Time independent perturbation Theory

Suppose we have solved the time-independent Schroding eqn for some potential,

$$\therefore\ H^0 \psi_n^0 = E_n^0 \psi_n^0 \quad \cdots\cdots (1)$$

obtaining a complete set of orthonormal eigenfunction say ψ_n^0,

$$\therefore\ \langle \psi_n^0 | \psi_m^0 \rangle = \delta_{nm} \quad \cdots (2)$$

and the corresponding eigenvalues E_n^0.

Now we perturb the potential slightly, ~~say ba~~ we would like to find the new eigenfunctions and eigenvalues,

$$H\psi_n = E_n \psi_n \quad \cdots\cdots (3)$$

Perturbation theory is a systematic procedure for obtaining approximate solutions to the perturbed problem, by building on the known exact solutions to unperturbed case.

To begin, we write new Hamiltonian as sum of two terms

$$H = H^0 + \lambda H' \quad \cdots (4)$$

H' is the perturbation, H^0 is unperturbed hamiltonian and $\lambda \to$ small number

H is true hamiltonian.

Next we write

$$\psi_n = \psi_n^0 + \lambda \psi_n^1 + \lambda^2 \psi_n^2 + \cdots \quad — (5)$$

$$E_n = E_n^0 + \lambda E_n^1 + \lambda^2 E_n^2 + \cdots \quad — (6)$$

E_n^1 is the first order correction, to n^{th} eigenvalue

ψ_n^1 is first order correction to n^{th} eigenfunction, and so on.

$$(H^0 + \lambda H')(\psi_n^0 + \lambda \psi_n^1 + \lambda^2 \psi_n^2 + \cdots) = (E_n^0 + \lambda E_n^1 + \lambda^2 E_n^2 + \cdots)(\psi_n^0 + \lambda \psi_n^1 + \cdots)$$

collecting powers

$$H^0\psi_n^0 + \lambda(H^0 \psi_n^1 + H'\psi_n^0) + \lambda^2(H^0 \psi_n^2 + H'\psi_n^1) + \cdots \quad — (7)$$

$$= E_n^0 \psi_n^0 + \lambda(E_n^0 \psi_n^1 + E_n^1 \psi_n^0) + \lambda^2(E_n^0 \psi_n^2 + E_n^1 \psi_n^1 + E_n^2 \psi_n^0) + \cdots \quad --- (8)$$