# ASUTOSH COLLEGE

## Department of Mathematics

## ASSIGNMENT

Subject Name: Mathematics (Honours)

Semester: 4th semester

Name of Teacher: Dr. Nandan Ghosh

**Name of Topic: Uniform Convergence of Sequence and Series of functions**

1. Show that the sequence $\{ n \log( 1+ \frac{x}{n}) \}$ is uniformly convergent on any closed bounded interval of form [0,b] but the convergence is not uniform on $[0,\infty)$.
2. Consider $f_n(x)= \frac{n^{\alpha} x}{1+x^2 n^{\beta}}$ with $\beta>\alpha \geq 0$. Find the relation between $\alpha$ and $\beta$ for which the sequence converges uniformly on [0,1].
3. Prove or disprove: If $\{ f_n\}$ and $\{g_n\}$ converges uniformly on **R** then $\{ f_n . g_n\}$ is also uniformly convergent on **R**.
4. Let $\{ fn\}$ be a sequence of continuous real valued functions that converges uniformly on the closed and bounded interval [a,b ] and let $F_n (x)= \int_a^x fn (t)dt$ , $a\leq x\leq b$. Show that $\{F_n\}$ converges uniformly on [a,b].
5. Examine the uniform convergence of the series $\sum_0^{\infty}(\frac{1}{Kx+2} - \frac{1}{Kx+x+2})$ , $0 \leq x \leq 1$.
6. Show that the series $\sum_{n=1}^{\infty} \frac{1+n^5}{1+n^7} (\frac{x}{3})^n$ is uniformly and absolutely convergent on [-3,3] .
7. Evaluate : $\lim_{x\to 0} \sum_{K=2}^{\infty} \frac{\cos Kx}{K(K+1)}$.
8. Show that the series $\sum_{n=1}^{\infty}(\frac{x}{x+1})^n$ is uniformly convergent on [0,a], a >0 . (Use Dini's theorem)
9. If $\{ fn\}$ be uniformly convergent sequence of functions from S to R and if g: E to R be uniformly continuous on $E \supset f_n (S)$ For each n, show that the composite functional sequence $\{g . f_n\}$ converges uniformly on S.
10. Suppose $f_n (x)$ is non zero for all x and n, converges uniformly on D and there exist a $\beta> 0$ such that $f(x)\geq \beta$ for all x in D. Show that $\{1/f_n\}$ converges uniformly to 1/f(x) on D.

**Note : Date of Submission of Assignment : 22/04/2020**