Kirchhoff's laws for AC circuit

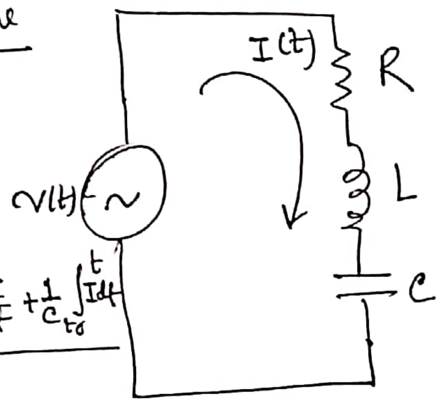
Kirchhoff's 1st law:- The algebric sum of the instantaneous currents flowing towards a junction is zero.

Kirchhoff's 2nd law:- The algebric sum of the instantaneous applied voltages in a closed loop equals the algebric sum of the instantaneous counter voltages in the loop.

Meaning of Kirchhoff's 1st law is if currents directed towards a junction are called positive, then those oppositely ~~directly~~ directed should be called negative. and the law says that as much current enters the junction as leaves it.

Kirchhoff's 2nd law represents the integral of the electric field around the loop. However, it is necessary to establish a sign convention. The sign convention to which we will adhere is best explained in terms of a single simple mesh.

Example



$\therefore V(t) = RI + L\frac{dI}{dt} + \frac{1}{C}\int_{t_0}^{t} I\,dt$

V(t) is connected in series with resistance 'R', inductance 'L' and capacitance 'C'.
The arrow labeled I(t) has been drawn to indicate the assumed (arbitrary) +ve dirn of current. All signs are referring to this dirn.

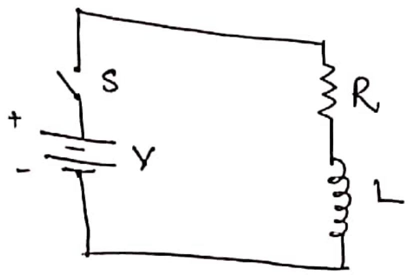
✓ V(t) is +ve if it tends to cause current to move in assumed direction. i.e if top terminal is +ve with respect to bottom terminal.

✓ Resistive counter voltage is 'IR', same as DC.

✓✓ If $\left(\frac{dI}{dt}\right) > 0$ an emf will be induced in inductance that tends to cause a current in opposite dirn to that assumed for 'I' — i.e upper terminal of L must be +ve with respect to lower terminal. i.e induced emf $= -L\left(\frac{dI}{dt}\right)$, but being emf it would normally written on other side of eqn. from counter voltages. Thus no inconsistency in writing $+L(dI/dt)$ for counter voltage.

✓✓✓ Capacitative counter voltage depends on charge on capacitor $Q = \int_0^t I(t)\,dt$

Transient Behavior (L-R) [Growth in L-R ckt]



$$V = RI + L\left(\frac{dI}{dt}\right)$$

or $$\left(\frac{dI}{dt}\right) + \frac{R}{L} I = \frac{V}{L}$$

$$I.F = e^{+\int_0^t \frac{R}{L} dt} = e^{+Rt/L}$$

$$\therefore \int d(I e^{+Rt/L}) = \int \frac{V}{L} e^{+Rt/L} dt$$

$$\Rightarrow I e^{+Rt/L} = +\frac{V}{\cancel{L}} \times \frac{\cancel{L}}{R} \cdot e^{+Rt/L} + C$$

At $t = t_0$, $I = 0$

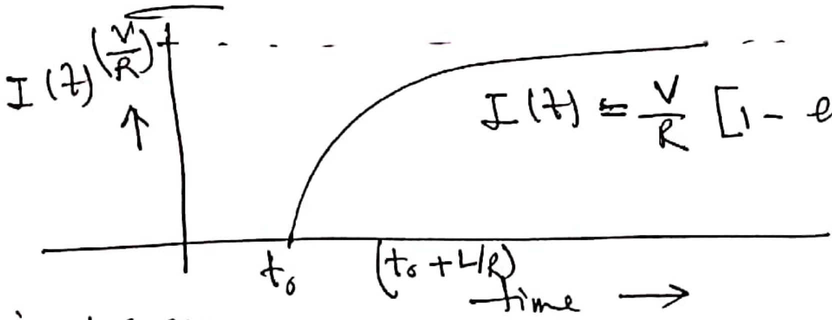
$$\therefore C = -V/R \; \exp(+Rt_0/L)$$

$$\therefore I(t) = +\frac{V}{R} - V/R \cdot \exp(+Rt_0/L) \cdot \exp\left(-\frac{Rt}{L}\right)$$

$$\therefore \boxed{I(t) = \frac{V}{R}\left[1 - e^{-\frac{R(t-t_0)}{L}}\right]}$$

$$\therefore \boxed{I(t) = \frac{V}{R}\left[1 - \exp\left\{-\frac{R(t-t_0)}{L}\right\}\right]}$$

'L/R' has dimension of "time" called time constant. Since, $1/e \cong 0.368$, i.e time constant is time required for current to reach 0.632 times final value.



$$I(t) = \frac{V}{R}\left[1 - \exp\left\{\frac{-R(t-t_0)}{L}\right\}\right]$$

Decay in L-R Ckt

Here, $RI + L\, dI/dt = 0$ i.e 'voltage source 'V' removed i.e. $V=0$

$$\therefore \int \frac{dI}{I} = \int -\frac{R}{L} dt$$

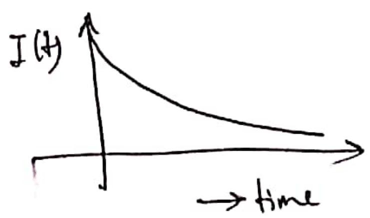
or $\ln(I) = -\frac{Rt}{L} + C$

at $t=0$, $I = I_0$

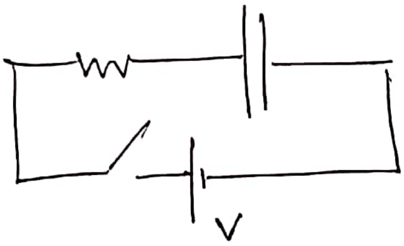
$\therefore C = \ln(I_0)$

$$\therefore \ln(I/I_0) = -Rt/L$$

$$\therefore \boxed{I = I_0 \exp(-Rt/L)}$$



C-R Circuit (Charging)



Let steady potential 'V' applied to ckt., 'i' be instantaneous value of current through 'R', potential at the plate of Condenser will be

$= (V - Ri)$

If 'q' is instantaneous charge on plate.

$$V = iR + \frac{\int i\,dt}{C}$$

or $\left(\frac{dq}{dt}\right) + \frac{q}{CR} = \frac{V}{R}$

Integrating factor $= e^{\int \frac{dt}{CR}} = e^{t/CR}$

$$\int d(q e^{t/CR}) = \int \frac{V}{R} e^{t/CR}\,dt$$

$\Rightarrow q e^{t/CR} = \frac{V}{\cancel{R}} \times C\cancel{R}\, e^{t/CR} + K$ (Integration Constant)

at, $t = 0$, $q = 0$

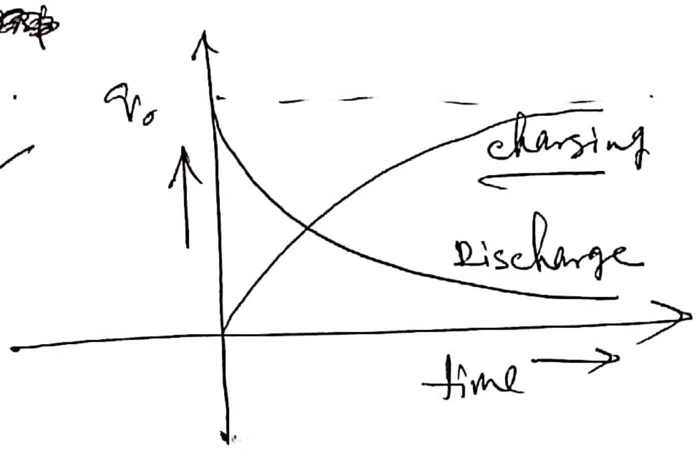
$\therefore K = -VC$ ~~$\exp(t/CR)$~~

$q e^{t/CR} = CV e^{t/CR} - VC$

$\therefore q = CV - CVe^{-t/CR}$.

$$\boxed{q = CV(1 - e^{-t/CR})}$$

$= q_0 \cdot (1 - e^{-t/CR})$



Similarly, (Discharging)

$\frac{dq}{dt} + \frac{q}{CR} = 0$

$\therefore \int \frac{dq}{q} = \int -\frac{dt}{CR}$

$\Rightarrow \ln q = -t/CR + K_1$

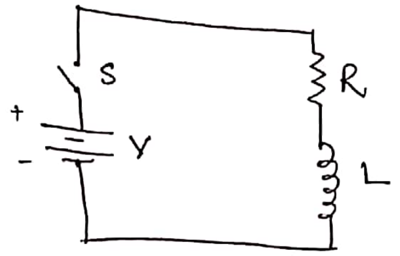
at $t = 0$, $q = q_0$

$\therefore K_1 = \ln(q_0)$

$\therefore \ln(q/q_0) = -t/CR$

$$\therefore \boxed{q = q_0 \exp(-t/CR)}$$

Transient Behavior (L-R) [Growth in L-R Ckt]



$$V = RI + L\left(\frac{dI}{dt}\right)$$

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At $t = t_0$, $I = 0$

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$$\therefore I(t) = +\frac{V}{R} - V/R \cdot \exp(+Rt_0/L) \cdot \exp\left(-\frac{Rt}{L}\right)$$

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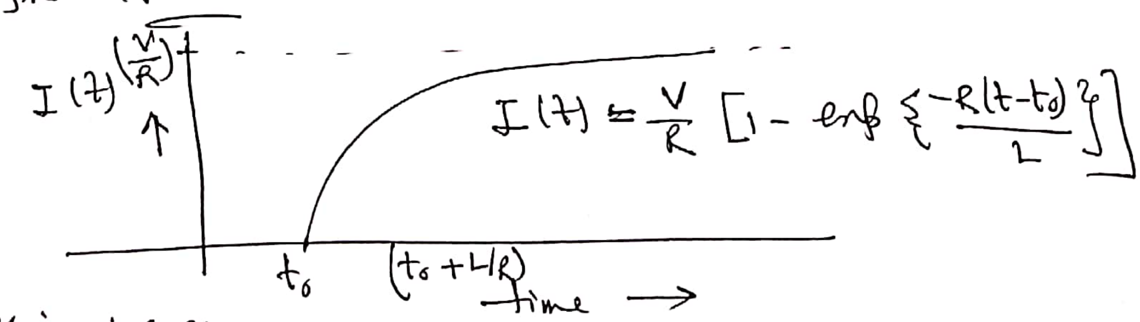

Decay in L-R Ckt

i.e 'voltage source 'V' removed i.e. $V = 0$

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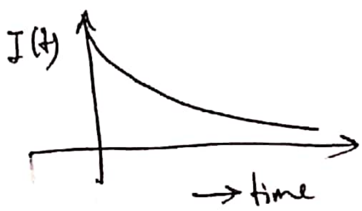

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