(a)

Feynman–Hellamann Theorem $\left[\text{To find } \langle \frac{1}{r} \rangle, \langle \frac{1}{r^2} \rangle\right]$

Let the hamiltonian 'H' depends on parameter 'λ', so that $H = H(\lambda)$. The eigenstates and eigenvalues of H are then also functions of 'λ', ie $E_n = E_n(\lambda)$ and $|n\rangle = |n(\lambda)\rangle$.

Now, we know.

$$\hat{H}|n\rangle = E_n|n\rangle$$

$$\therefore E_n = \langle n|H|n\rangle$$

Differentiating above eqn. w.r.t. 'λ' gives,

$$\frac{\partial E_n}{\partial \lambda} = \left(\frac{\partial}{\partial \lambda}\langle n|\right) H|n\rangle + \langle n|\left(\frac{\partial H}{\partial \lambda}\right)|n\rangle + \langle n|H\frac{\partial}{\partial \lambda}|n\rangle$$

Since, $H|n\rangle = E_n|n\rangle$

and $\langle n|H = \langle n|E_n$.

$$\therefore \frac{\partial E_n}{\partial \lambda} = E_n\left(\frac{\partial}{\partial \lambda}\langle n|\right)|n\rangle + \langle n|\left(\frac{\partial}{\partial \lambda}|n\rangle\right) + \langle n|\frac{\partial H}{\partial \lambda}|n\rangle$$

$$= E_n\frac{\partial}{\partial \lambda}\langle n|n\rangle + \langle n|\frac{\partial H}{\partial \lambda}|n\rangle$$

Since, $\langle n|n\rangle = 1$

$$\therefore \boxed{\frac{\partial E_n(\lambda)}{\partial \lambda} = \langle n(\lambda)|\frac{\partial H(\lambda)}{\partial \lambda}|n(\lambda)\rangle}$$

$\rightarrow$ This is Feynman-Hellmann Theorem