Deformation and Strain

Deformation of a body occurs in response to forces

Deformation describes the collective displacements of points in a body;

or

the complete transformation from the initial to the final geometry of a body.

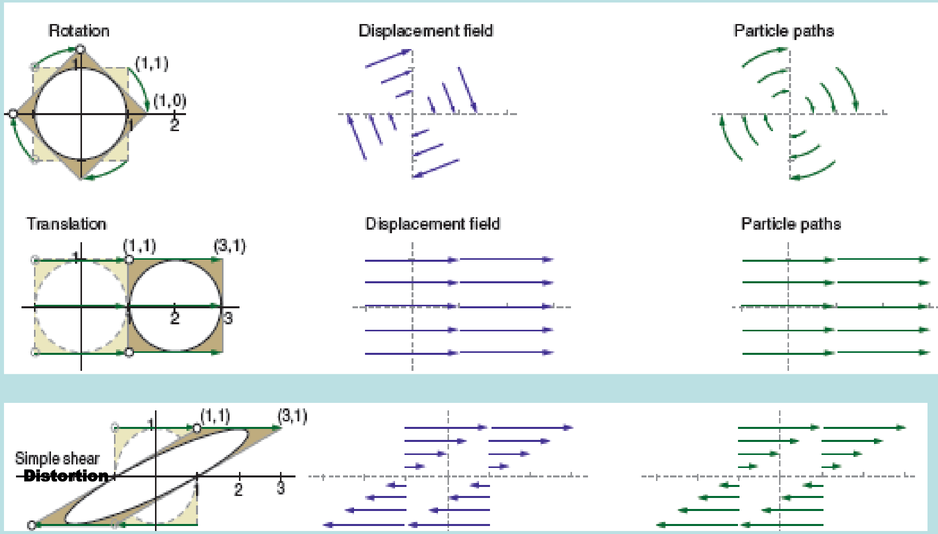
Deformation is the transformation from an initial to a final geometry by means of rigid body translation, rigid body rotation, strain (distortion) and/or volume change.

This change can include

a **translation** (movement from one place to the other)

a **rotation** (spin around an axis),

and a **distortion**



Translation moves every particle in the rock in the same direction and the same distance, and its displacement field consists of parallel vectors of equal length.
The rotation and distortion components are zero, we have only rotation of the body.

Rigid rotation involves a uniform physical rotation of a rock volume relative to an external coordinate system.
The translation and distortion components are zero, we have only rotation of the body.

Deformation relates the positions of particles before and after the deformation

The positions of points before and after deformation can be connected with vectors.

These vectors are called displacement vectors, and a field of such vectors is referred to as the displacement field

The actual path that each particle follows during the deformation history is referred to as a particle path

The translation and rotation components are zero, we have only rotation of the body.

Deformation describes the complete displacement field of points in a body relative to an *external reference frame*

While

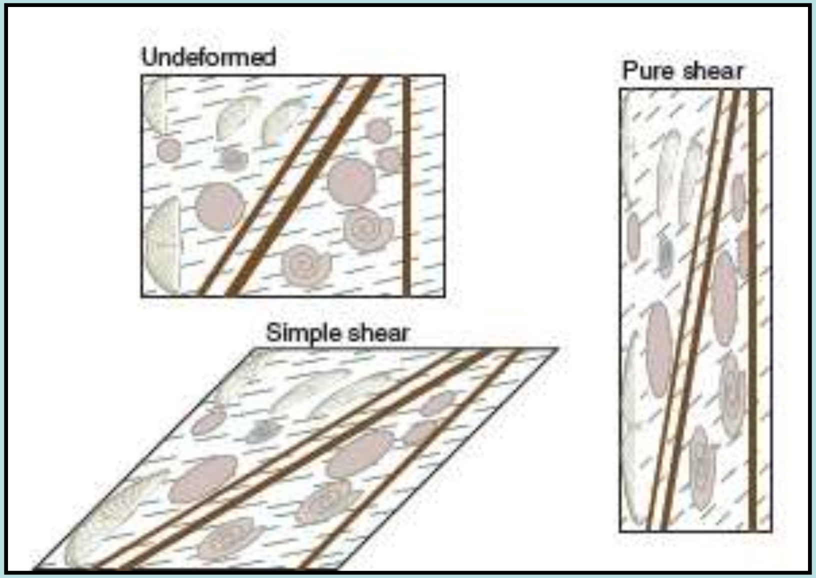
Strain describes the displacement field of points relative to each other. This requires a reference frame within the body, an *internal reference frame,* like the edges of the square.

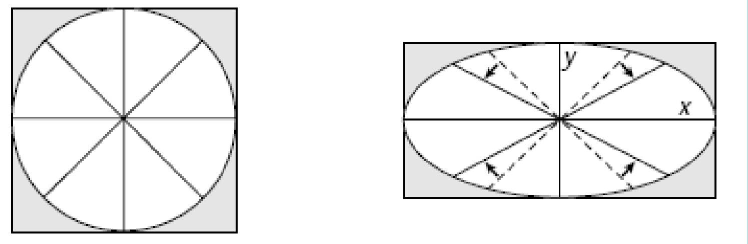
Strain: Any change in shape, with or without change in volume, is referred to as strain, and it implies that particles in a rock have changed positions relative to each other

Changes in lengths of lines
Changes in volume
Changes in angles

Homogeneous strain: Strain is **homogeneous** when any two portions of the body that were similar in form and orientation before are similar in form and orientation after strain.

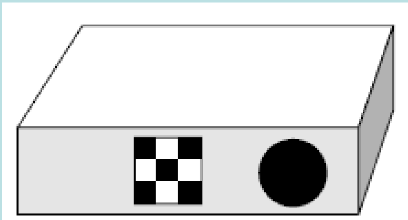
Straight lines remain straight
Parallel lines remain parallel,
Identically shaped and oriented objects will also be identically shaped and oriented after the deformation
(Circles become ellipses; in three dimensions, spheres become ellipsoids)

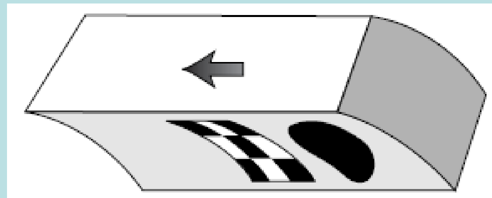


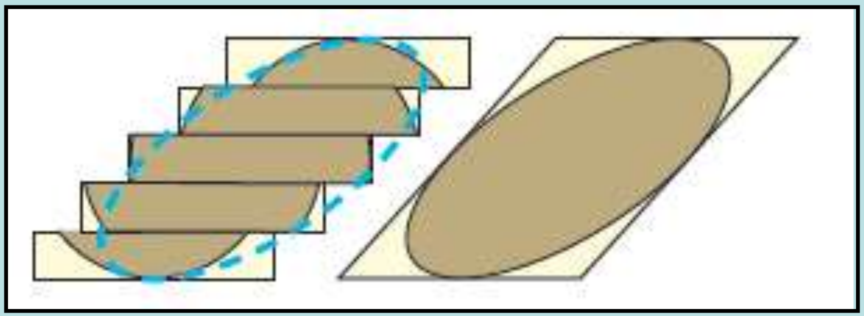


Homogeneous strain describes the transformation of a square to a rectangle or a circle to an ellipse.
Two material lines that remain perpendicular before and after strain are the principal axes of the strain ellipse (solid lines).
The dashed lines are material lines that do not remain perpendicular after strain; they rotate toward the long axis of the strain ellipse.

If not homogeneous, then the deformation is heterogeneous (inhomogeneous).







Discrete or discontinuous deformation can be approximated as continuous and even homogeneous in some cases.

In this sense the concept of strain can also be applied to brittle deformation (brittle strain). The success of doing so depends on the scale of observation

Longitudinal strain is defined as a change in length divided by the original length. Longitudinal strain is expressed by the **elongation, e,** which is defined as

$\mathbf{e} = (l - l_o)/l_o$ or $\mathbf{e} = \delta l/l_o$

where l is the final length, l_o is the original length, and δl is the length change

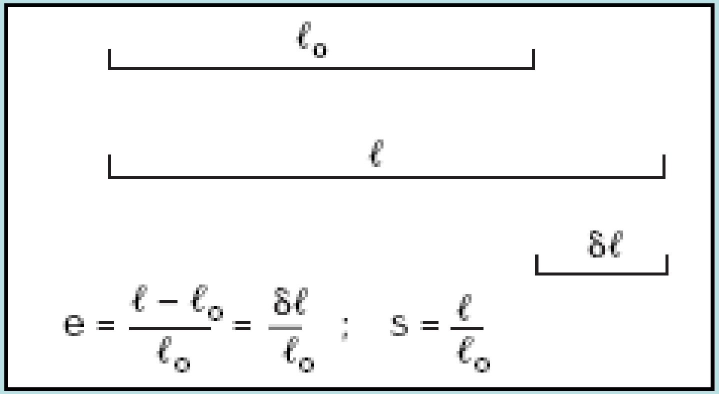
Quadratic elongation, λ, which is defined as

$\lambda = (l/l_o)2 = (1 + \mathbf{e})^2$

where l is the final length, l_o is the original length, and **e** is the elongation.

The root of the quadratic elongation is called the **stretch, s,** which is a convenient strain parameter that directly relates to the dimensions of the strain ellipsoid:

$\mathbf{s} = \lambda 1/2 = l/lo = 1 + \mathbf{e}$



The quadratic elongation, λ, and especially the stretch, **s,** are convenient measures because they describe the lengths of the principal axes (*X*, *Y,* and *Z*) of the strain ellipsoid:

$X = \mathbf{s}1,\ Y = \mathbf{s}2,\ Z = \mathbf{s}3$

with $X \geq Y \geq Z$, and

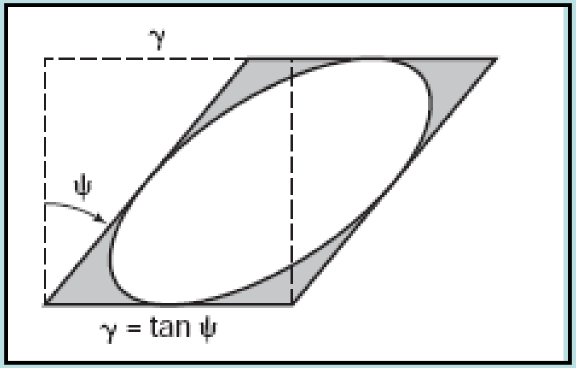
$X2 = \lambda 1,\ Y2 = \lambda 2,\ Z2 = \lambda 3$

For *volumetric strain,* Δ, the relationship is $\Delta = (V - V_o)/V_o$ or $\Delta = \delta V/V_o$

where V is the final volume, V_o is the original volume, and δV is the volume change.

The change in angle is called the **angular shear,** ψ, but more commonly we use the tangent of this angle, called the **shear strain,** γ

$\gamma = \tan \psi$

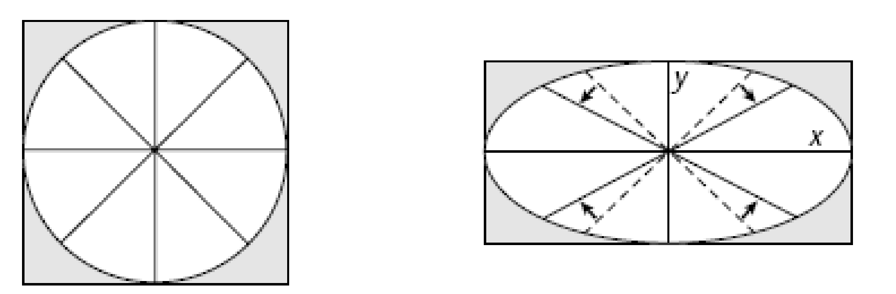


In **homogeneous** strain, two orientations of material lines remain perpendicular before and after strain. These two material lines form the axes of an ellipse that is called the **strain ellipse.**

lengths of these two material lines change from the initial to the final stage.

In three dimensions we have three material lines that remain perpendicular after strain and they define the axes of an ellipsoid, the **strain ellipsoid.**

The lines that are perpendicular before and after strain are called the **principal strain axes.** Their lengths define the strain magnitude and we will use the symbols *X, Y,* and *Z* to specify them, with the convention that $X \geq Y \geq Z$.

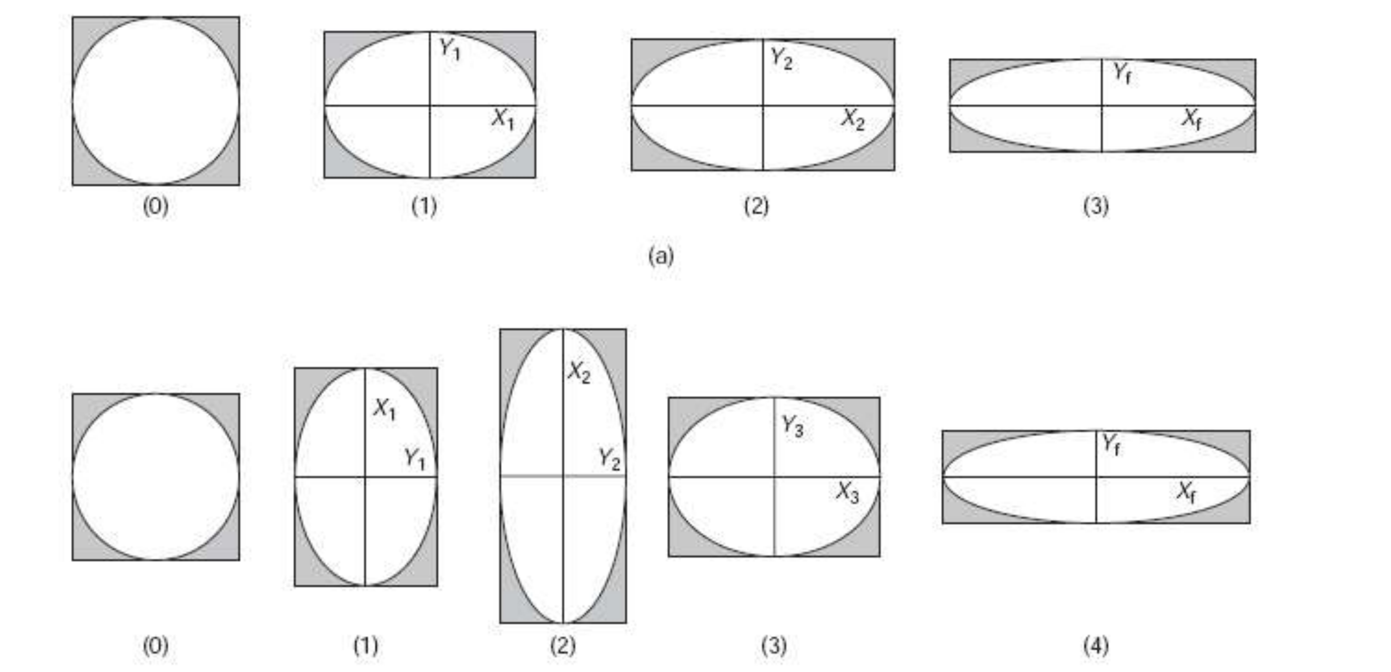


Homogeneous strain describes the transformation of a square to a rectangle or a circle to an ellipse. Two material lines that remain perpendicular before and after strain are the principal axes of the strain ellipse (solid lines).

The measure of strain that compares the initial and final configuration is called the **finite strain,** which is independent of the details of the steps toward the final configuration.

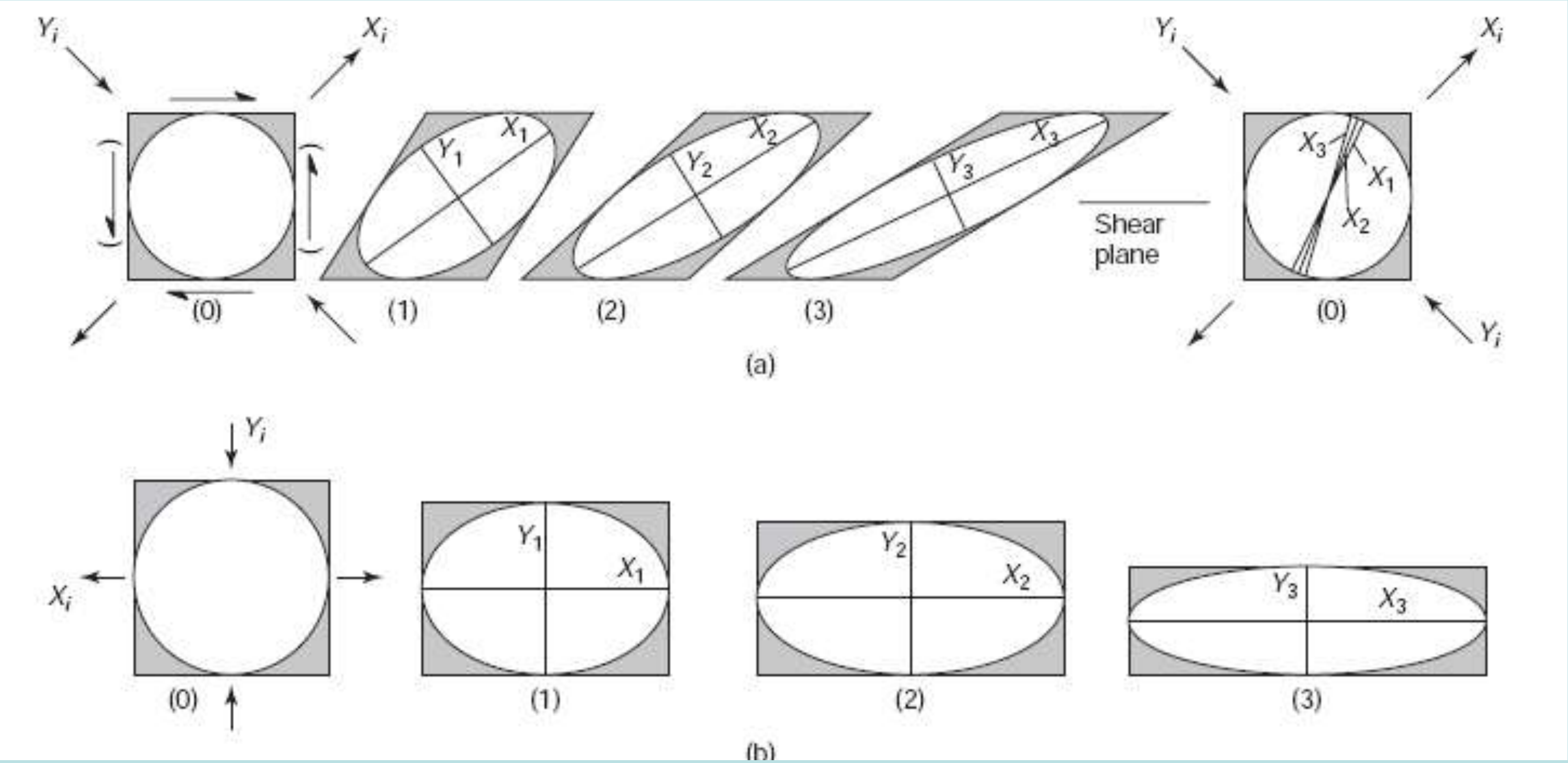
When these intermediate strain steps are determined they are called **incremental strains**.

The summation of all incremental strains (that is, product), therefore, is the finite strain



The principal incremental strain axes rotate relative to the finite strain axes, a scenario that is called non-coaxial strain accumulation.

The case in which the same material lines remain the principal strain axes at each increment is called coaxial strain accumulation.



The finite spatial change in shape that is connected with deformation is completely described by the strain ellipsoid.

The strain ellipsoid is the deformed shape of an imaginary sphere with unit radius that is deformed along with the rock volume under consideration.

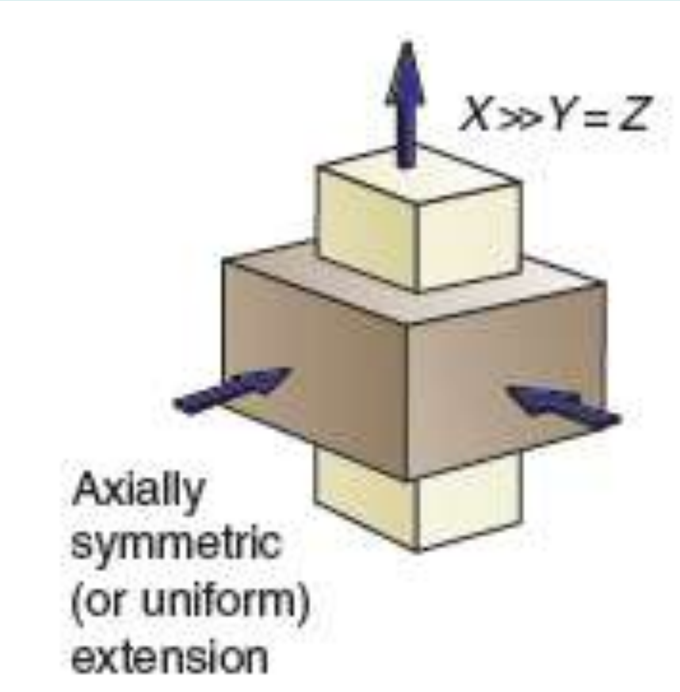
The strain ellipsoid has three mutually orthogonal planes of symmetry, the principal planes of strain, which intersect along three orthogonal axes that are referred to as the principal strain axes.

Their lengths (values) are called the principal stretches. These axes are commonly designated X, Y and Z, but the designations √l1, √l2 and √l3, S1, S2 and S3 as well as e1, e2 and e3 are also used.

X > Y > Z

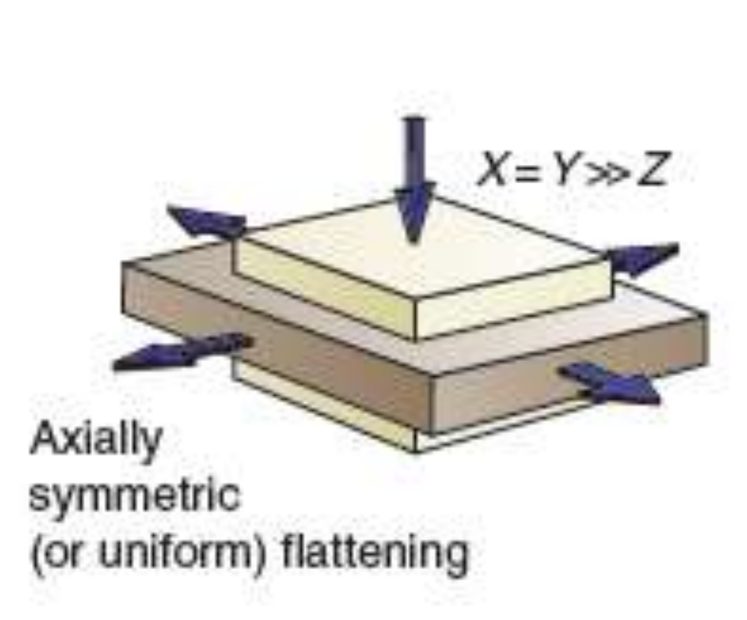
When the ellipsoid is fixed in space, the axes may be considered vectors of given lengths and orientations. Knowledge of these vectors thus means knowledge of both the shape and orientation of the ellipsoid. The vectors are named e1, e2 and e3, where e1 is the longest and e3 the shortest

Uniform extension, also referred to as axially symmetric extension, is a state of strain where stretching in X is compensated for by equal shortening in the plane orthogonal to X.



Uniform flattening (axially symmetric flattening) is the opposite, with shortening in a direction Z compensated for by identical stretching in all directions perpendicular to Z.

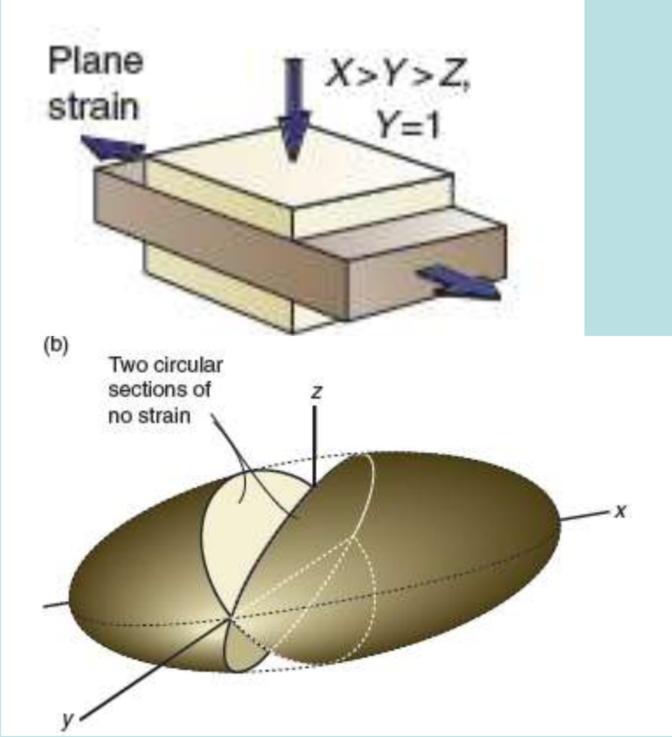
These two reference states are end-members in a continuous spectrum of
deformation types.



Between uniform flattening and extension lies **plane strain**, where stretching in one direction is perfectly compensated by shortening in a single perpendicular direction.

The strain is “plane” or two-dimensional because there is no stretching or shortening in the third principal direction, i.e. along the Y-axis.

Within any body that has undergone a plane strain, there are always two lines of no finite stretch along which there has been neither lengthening or shortening; lines: oriented in these directions are characterized by stretch values of 1.0.

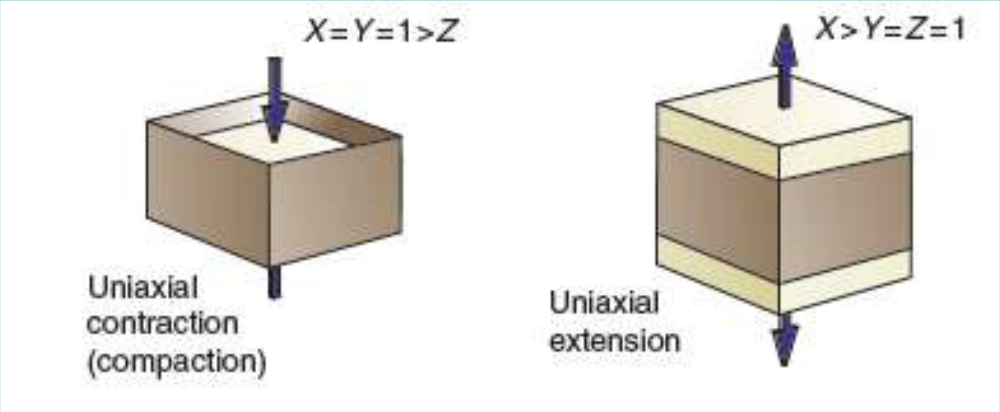


Strain is said to be plane (two-dimensional) where there is no length change along the Y-axis, while three-dimensional strain implies a length change along X and Z.

A plane strain deformation produces two planes in which the rock appears unstrained.

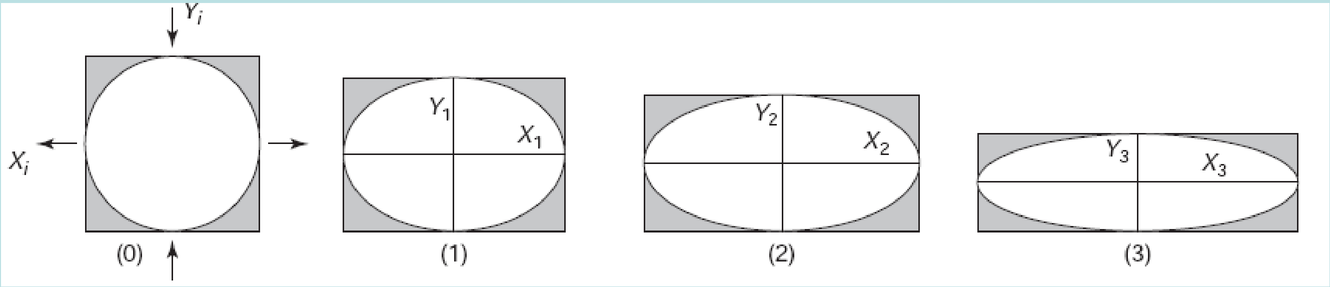
Uniaxial extension implies expansion in one direction. This may occur by the formation of tensile fractures or veins or during metamorphic reactions.

Uniaxial contraction: X=Y>Z, X=1
Uniaxial extension: X>Y=Z, Z=1

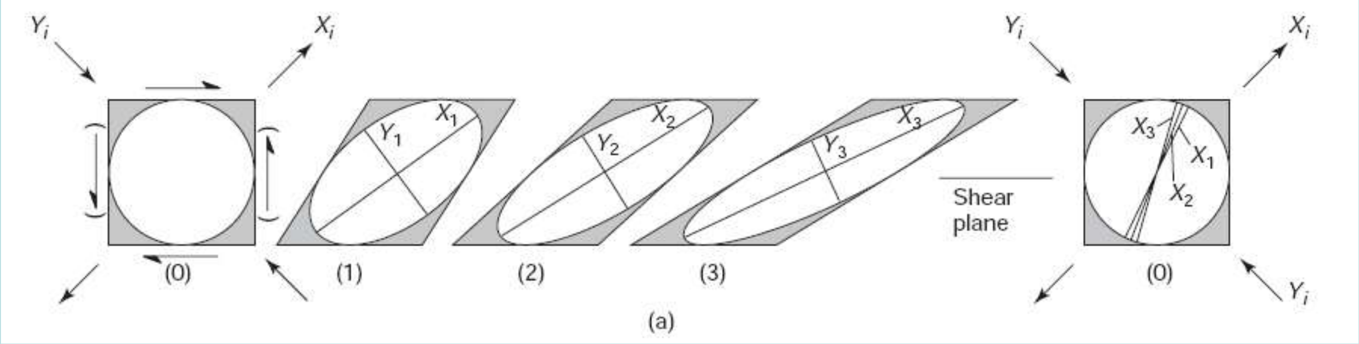


The component describing the rotation of material lines with respect to the principal strain axes is measure of the degree of non-coaxiality.

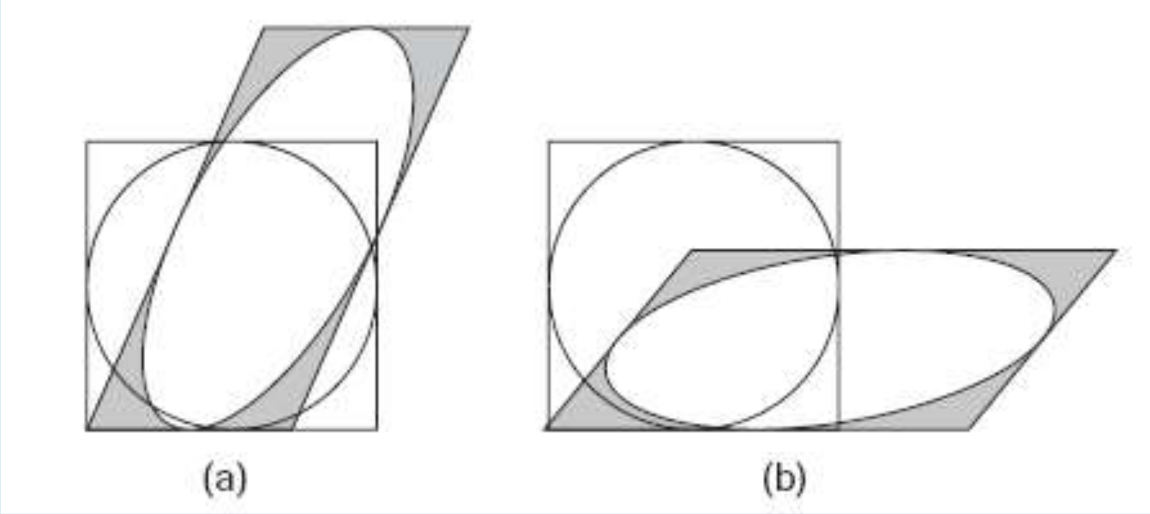
If there it is zero, the strain history is coaxial, which is sometimes called **pure shear.**

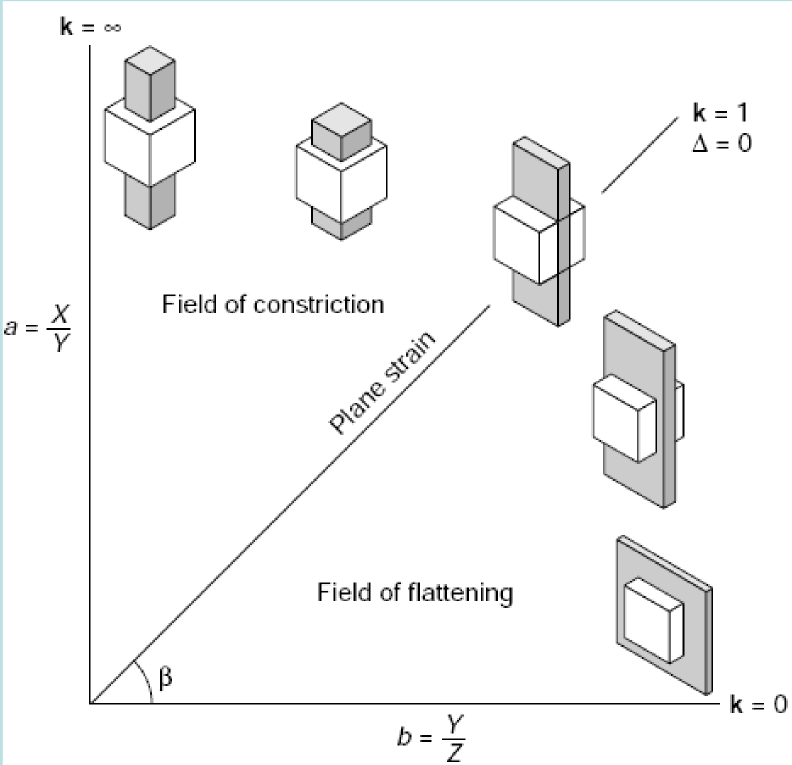


The non-coaxial strain history describes the case in which the distance perpendicular to the shear plane (or the thickness of our stack of cards) remains constant; this is also known as **simple shear.**



In reality, a combination of simple shear and pure shear occurs, which we call **general shear** or **general non-coaxial strain accumulation;**





$$a = X/Y = (1 + \mathbf{e}_1)/(1 + \mathbf{e}_2)$$
$$b = Y/Z = (1 + \mathbf{e}_2)/(1 + \mathbf{e}_3)$$

$$\mathbf{k} = (a - 1)/(b - 1)$$

A strain sphere lies at the origin of this plot (coordinates 1,1), representing $a = b = 1$ or $X = Y = Z = 1$.

Ellipsoid shapes are increasingly **oblate** for values of **k** approaching 0 and increasingly **prolate** for values of **k** approaching α.
If **k** = 0, the strain is uniaxially oblate ($a = X/Y = 1$), and if **k** = αα, the strain is uniaxially prolate ($b = Y/Z = 1$). The value
k = 1 represents the special case for which *a* equals *b,* which is called plane strain ($X \geq Y = 1 \geq Z$). The line represented by **k** = 1 states separates the field of *constriction* (∞ > **k** > 1) from the field of *flattening* (1 > **k** > 0) in the Flinn diagram.

The shape of the strain ellipsoid is represented by the parameter, **k**