2021

ASUTOSH COLLEGE

Department of Mathematics

Subject: Mathematics General (MTMG) **Semester: VI**

Time: 1 hour **Paper: DSE-B (Advanced Calculus)** **F.M- 25**

SET-I

INTERNAL (5x2=10 MARKS)

Choose the correct option from below with a proper justification

[Answer all questions. Each question carry 2 marks. 1 mark for correct option & 1 mark for justification]

1.The radius of convergence of the power series $x + \frac{2^2x^2}{2!} + \frac{3^3x^3}{3!} + \cdots$ is

a) e b) $\frac{1}{e}$ c) 1 d) e^2

2.The limit function $f(x)$ of the sequence of function $\{f_n\}$, $f_n : \mathbb{R} \to \mathbb{R}$ defined by $f_n(x) = \frac{nx}{1+n^2x^2}$ is

a) 0 b) 1 c) 2 d) None of these

3. If $\sum a_n x^n$ be a power series and let $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \mu$, then which of the following value of μ states that the series is everywhere convergent

a) $\mu = 0$ b) $\mu = 1$ c) $\mu = \infty$ d) $\mu = 2$

4. Solution of y''+y=1 using Laplace Transform, with given conditions y '(0)=1,y(π)=0 is

a) y(t)= cost+sint b) y(t)= cost+sint+1

c) y(t)= cost+sint -1 d) None of these

5. $L\{(1 + te^{-t})^3\}$ is

a) $\frac{1}{p} + \frac{6}{(p+3)^4} + \frac{6}{(p+2)^3} + \frac{3}{(p+1)^2}$ b) $\frac{1}{p} + \frac{6}{(p+3)^4} + \frac{6}{(p+2)^3} + \frac{6}{(p+1)^2}$

c) $\frac{1}{p} + \frac{6}{(p+3)^4} + \frac{3}{(p+2)^3} + \frac{2}{(p+1)^2}$ d) None of these

TUTORIAL (5x3=15 marks)

[Answer any five (5) questions from below. Each question carry 3 marks]

6. Find $\lim_{x\to 0} \sum_{n=1}^{\infty} \frac{\cos nx}{n(n+1)}$

7. Let $f(x)$ be the sum function of the power series $\sum_{n=0}^{\infty} a_n x^n$ on (-R,R) for some R>0. If $f(x) = f(-x)$ for all $x \in (-R, R)$. Show that $a_n = 0$ for all odd n.

8. A function f is defined on $\left(-\frac{1}{3}.\frac{1}{3}\right)$ by $f(x) = 1 + 2.3\,x + 3.3^2x^2 + \cdots n.3^{n-1}x^{n-1} + \cdots$ Show that f is continuous on $\left(-\frac{1}{3}, \frac{1}{3}\right)$. Evaluate $\int_0^{\frac{1}{4}} f$.

9. Find Inverse Laplace transform of $\frac{p}{(p^2+4)^3}$.

10. Expand in fourier series $x+x^2$ on $[-\pi,\pi]$ and deduce that $\frac{\pi^2}{6}=1+\frac{1}{2^2}+\frac{1}{3^2}+\ldots$.

11. Use convolution Theorem and find $L^{-1}\left\{\frac{p^2}{(p^2+4)^3}\right\}$.
