2021

PHYSICS — HONOURS

Paper : DSE-B-2

(Advanced Statistical Mechanics)

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group - A

1. Answer ***any five*** questions : 2×5

(a) Consider a system with N particles and three energy states. The states are $E_1 = 0$, $E_2 = K_BT$ and $E_3 = 2K_BT$. The total energy of the system is 3000 K_BT. Find the number of particles.

(b) Classify the following particles according to Fermi or Bose statistics :

(i) ^{3}He, (ii) H_2 molecule, (iii) $^6Li^+$ion, (iv) $^7Li^+$ion

(c) A classical gas of molecules, each of mass m, is in thermal equilibrium at absolute temperature T. If v_x, v_y and v_z are the three cartesian components of the velocity of a molecule, then calculate $\langle (v_x - v_y)^2 \rangle$.

(d) The Hamiltonian of a system in 1–D is $H = ap_x{}^2 + bx^2 + cx$. Find the value of $\langle H \rangle$.

(e) Show that the density operator for a pure state is idempotent.

(f) A random walker takes a step of unit length in the positive direction with the probability $^2/_3$ and a step of unit length in the negative direction with probability $^1/_3$. Calculate the mean displacement of the walker after n-step.

(g) What do you mean by coarse graining in non-equilibrium statistical mechanics?

Group - B

Answer ***any three*** questions.

2. Using the fact that the Gibbs free energy $G(N, p, T)$ of a thermodynamic system is an extensive property of the system, show that $G = N\frac{\partial G}{\partial N}$. Hence show that $G = N\mu$, where μ represents the chemical potential of the system. 5

3. Calculate the value of $-\frac{\partial f(\epsilon)}{\partial \epsilon}$ at $\epsilon = \mu$, where $f(\epsilon)$ represents the FD distribution function. Show that at $T \to 0, -\frac{\partial f(\epsilon)}{\partial \epsilon}$ at $\epsilon = \mu$ behaves as a delta function. 2+3

4. Consider free electrons in silver (Ag) at 300 K. Test whether it is a classical system or not.

Given : Density of Ag = 10·5 gm/cc.

Atomic weight = 107·9 gm.

Boltzmann constant $k_B = 1\cdot381\times10^{-23}\ J/K$. 5

5. Consider N distinguishable and non-interacting particles. The single particle energy spectrum is $\epsilon_n = n\epsilon$, with $n = 0, 1, 2, ..., \infty$ and degeneracy $g_n = n + 1$ ($\epsilon > 0$ is a constant). Compute the canonical partition function and the average energy. 5

6. A system has two spin states $S = +1$ or $S = -1$. The system contains N number of molecules. If there is interaction among the particles write down the Hamiltonian using Ising model. Now using Bragg Williums approximation show that the Hamiltonian can be expressed as

$$-\frac{1}{2}\gamma\,\epsilon\,Nm^2 - \mu BmN. \qquad 5$$

where m is the long range order parameter, γ is the number of nearest neighbour of the particle, ϵ is the interaction energy and μ is the magnetic moment of the particle.

Group - C

Answer ***any four*** questions.

7. A system has two energy levels with energy 0 and ϵ. The system may be either unoccupied or occupied by a single particle in any one of its energy level. Calculate the grand canonical partition function of the system. Determine the average occupancy $\langle N\rangle$ of the system. Find an expression for thermal average energy of the system. 4+3+3

8. (a) Consider a spin $\frac{1}{2}$ system with a pure state

$$|\alpha\rangle = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

(i) find corresponding density matrix

(ii) find the density matrix for 'up' spin state $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

(b) The Hamiltonian of an electron in a magnetic field $\vec{B}$ is given by $H = -\mu_B\vec{\sigma}\cdot\vec{B}$, where $\vec{\sigma}$ is Pauli spin operator and μ_B stands for the Bohr magneton. Construct the density matrix in the diagonalized representation of σ_z. Also calculate the $\langle\sigma_z\rangle$. the z-axis being taken along the field direction.

(2+2)+3+3

9. (a) Show that the pressure of the weakly degenerate bosons is less than that of the classical gas. [Given the relation

$$pV = Nk_B T \frac{g_{5/2}(z)}{g_{3/2}(z)}$$

where

$$\left. g_\nu(z) = \sum_{n=1}^{\infty} \frac{z^n}{n^\nu} = z + \frac{z^2}{2^\nu} + \frac{z^3}{3^\nu} + \ldots \right]$$

(b) Experimentally Bose–Einstein condensation was produced in a vapour of ^{87}Rb atoms at a number density of $2{\cdot}5\times10^{12}\ cm^{-3}$. Calculate the critical temperature.

(c) Explain why Bose condensation is not possible in one dimension. 3+3+4

10. (a) Consider a gas of free electrons confined inside a three-dimensional box. The z-component of the magnetic moment of each electron is $\pm\,\mu_B$. In the presence of a magnetic field B pointing in the z-direction, each 'up' state acquires an additional energy of $-\,\mu_B B$, while each 'down' state acquires an additional energy of $+\,\mu_B B$.

(i) Explain why you would expect the magnetization of a degenerate electron gas to be substantially less than that of the paramagnets for a given number of particles at a given field strength.

(ii) Obtain an expression for the net magnetization of this system at $T = 0$.

(b) Consider a degenerate electron gas in which essentially all of the electrons are highly relativistic ($\epsilon >> mc^2$), so that their energies are $\epsilon = pc$. Calculate the chemical potential of such a system. (2+5)+3

11. (a) Calculate the fluctuations $\langle N^2\rangle - \langle N\rangle^2$ in number density N in grand cannonical ensemble. Hence evaluate the relative root mean square fluctuation in N.

(b) Obtain the partition function of an ideal gas in grand cannonical ensemble. Hence calculate the internal energy and grand potential of the system. (4+1)+(2+2+1)

12. (a) Estimate the Fermi energy and Fermi temperature of an white dwarf star. Given : mass of proton $9{\cdot}1\times10^{24}$ gm, mass density of star $\sim 10^7$ g/cc.

(b) Explain why the electron gas in white dwarf star is highly degenerate.

(c) Consider a person walking randomly in 1D from a point. Find the probability that the person after N displacement will be at a distance $x = ml$, where m is an integer and l is the step size. 10

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2021

PHYSICS — HONOURS

Paper : DSE-B2(a)

(Communication Electronics)

Full Marks : 65

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Candidates are required to give their answers in their own words as far as practicable.

Group - A

1. Answer ***any five*** questions : 2×5

(a) What is amplitude modulation? Write down the expression of an AM wave.

(b) Calculate the power developed by an AM wave across a load of 100 Ω when the peak voltage of the carrier is 100 Volts and modulation index is 0.4.

(c) What is the need for modulation in case of communication?

(d) Find the Nyquist rate and Nyquist interval for the continuous time signal given below.

$$x(t) = \cos(4000\,\pi t)\ \cos(1000\,\pi t)$$

(e) An amplifier has an input of 3 mV and output of 5V. What is the gain of the amplifier in dB?

(f) Find the bandwidth of 8-PSK.

(g) What is SIM and IMEI no. in mobile communication?

Group - B

Answer ***any three*** questions.

2. A 1 MHz carrier wave is amplitude modulated by audio signals between 400 Hz to 1600 Hz. Find (i) the frequency span of each side band, (ii) the maximum upper side band frequency, (iii) the maximum lower side band frequency and (iv) the channel width. 1+1+1+2

3. (a) Show that the total power for a fully amplitude modulated wave is 1·5 times the unmodulated carrier power.

(b) Write down differences between amplitude modulation (AM) and frequency modulation (FM). 2+3

4. Describe sampling theorem. What is quantization error? Differentiate between impulse, natural and flat-top sampling. 1+1+3

5. How can non-uniform quantization be used to increase SNR? Draw the constellation diagram for Q-PSK. 2+3

6. What is path loss of satellite communication system? How is the path loss related to the gain and power of the transmitting and receiving antenna? 2+3

Group - C

Answer ***any four*** questions.

7. (a) What is frequency modulation? Show that the maximum deviation from the carrier frequency is independent of modulating frequency.

(b) A frequency modulated wave is represented by

$$v_{FM} = V_c \cos\left(\omega_c t + m_f \sin \omega_m t\right)$$

where the symbols have their usual meaning. Show that if the modulation index $m_f \ll 1$, then the bandwidth of the FM is approximately $2\,\omega_m$.

(c) What is thermal noise? Calculate the thermal noise voltage developed across a resistor of 700 Ω. The bandwidth (B.W.) of measuring instrument is 7 MHz and the ambient temperature is 27ºC. (Given Boltzmann constant $k = 1{\cdot}38\times10^{-23}$ J/K) (1+2)+3+(1+3)

8. (a) What is meant by the terms modulation efficiency and percentage modulation in AM?

(b) Determine the modulation index and the percentage of total power carried out by the side bands of the AM wave for the modulation, when the modulation index is 0·5. 5+5

9. (a) Describe FDM and TDM.

(b) What is aliasing? What can be done to reduce aliasing?

(c) What type of filter is used in TDM receiver? (2½+2½)+(2+2)+1

10. (a) Describe the basic principles of satellite communication.

(b) What are the differences among 2G, 3G and 4G technologies in mobile communication system?

(c) What is geostationary satellite? State the advantages of it. 4+3+(2+1)

11. (a) What is ASK? Explain the function of FSK graphically when the digital input message signal is 011001.

(b) What are the possible levels for BPSK and QPSK?

(c) Describe A-Law for companding. Define unipolar and bipolar RZ and NRZ. (2+3)+2+(1+2)

12. (a) What do you mean by transponder in satellite communication? What are their basic components?

(b) Draw the block diagram of Earth station.

(c) Write down the three basic segments in GPS. What is the basic performance of Master Control System (MCS) in GPS? (2+2)+3+(1+2)
