2021

STATISTICS — GENERAL

Paper : SEC-B-2

(Monte Carlo Methods)

Full Marks : 80

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group - A

Answer ***any fifteen*** questions 2×15

1. What is Pseudo Random Number Generator (PRNG)?

2. State the Inverse Transform Method for generating random variates.

3. Describe a process of generating a random variable such that $p_1 = 0{\cdot}25$ and $p_2 = 0{\cdot}75$, where $p_i = Pr(X = i)$, $i = 1,\ 2$.

4. How will you simulate n Bernoulli variables with parameter p?

5. Generate a random variable with p.d.f. $f(x) = \begin{cases} 2x & \text{if } 0 \le x \le 1 \\ 0 & \text{otherwise.} \end{cases}$

6. Generate a random variable with uniform p.d.f. $f(x) = \begin{cases} \dfrac{1}{b-a} & \text{if } a \le x \le b \\ 0 & \text{otherwise} \end{cases}$.

7. Describe a method of evaluating the following integral by Monte Carlo Method :

$$\int_0^1 e^x dx.$$

8. Describe a method of generating a random variable with distribution function $F(x) = x^n$, $0 \le x \le 1$.

9. Mention any two desirable properties of a good random number generator.

10. Given a random observation from $N(0, 1)$, how can you generate an observation from $N(5, 2^2)$?

11. What do you mean by the period length of a random number generator?

12. Suppose you are given a random observation from a standard normal distribution. How do you generate one observation from a χ^2 distribution with 1 degree of freedom?

13. Suppose you are given a random observation from $N(\mu, \sigma^2)$. How can you generate an observation from a log-normal distribution with parameters μ and σ^2?

14. A number U is generated at random from the interval [0, 1]. Write down the distribution function of $X = U^2$.

15. Why are computer generated random numbers called pseudorandom numbers?

16. Why the initial seed x_0 must not be equal to 0 or m in a multiplicative congruential generator

$x_{i+1} = ax_i \ mod \ m, i \geq 0$?

17. Consider the random number generator $x_n = 5x_{n-1}$ mod 2^5 with $x_0 = 2$. After how many numbers, the seed x_0 will appear again?

18. Given a random observation 0.78 from $U(0, 1)$, simulate an observation from the distribution having p.d.f. $f(x) = e^{-x};\ x > 0$.

19. Consider a biased coin that produces heads 70% of the time. Given a random observation 0.63 from $U(0, 1)$, what would be the likely outcome in a single toss of the coin?

20. Suppose you are given a biased six faced die, where the probability of obtaining any of the faces 1, 2,..., 5 are equally likely and 6 appears with probability 0.5. Given one observation from $U(0, 1)$ as 0.45, what would be the likely outcome in a single throw of the die?

Group B

Answer ***any six*** questions 5×6

21. How will you estimate π by applying the Monte Carlo method?

22. Write a short note on Importance Sampling.

23. Describe a process of generating n random variables from an exponential distribution with mean λ.

24. Describe a method for generating a geometric random variable X with parameter p.

25. Describe the Box-Muller method for generating standard normal variate.

26. Discuss how you will find expectation of a random variable $X \sim Beta(1, 4)$ by Monte Carlo method.

27. How can you generate a sequence of pseudorandom numbers using the linear congruential generators?

28. Describe a method to simulate the roll of a fair die.

Group C

Answer ***any two*** questions 10×2

29. Let $X_1, X_2, ..., X_n$ be i.i.d. random variables following exponential distribution with mean $1/\mu$. Define $Y_1 = min\{X_1, ..., X_n\}$ and $Y_n = \max\{X_1, ..., X_n\}$. Generate Y_1 and Y_n by using the Monte Carlo method.

30. Describe with an algorithm how you will simulate a random permutation of 1, 2, .., n.

31. Let g be a function defined on [–2, 2] given by

$$g(x) = \frac{8}{7} + \frac{118}{63}x^2 - \frac{74}{63}x^4 + \frac{10}{63}x^6.$$

Find c such that $f(x) = c.g(x)$ is a probability density function. Describe a method of generating a sample from f.