<u>Assignment</u>

Subject: Mathematics

Semester: 2^{nd}

Name of Teacher: Prabir Rudra

Topic: Cosets, Lagrange's Theorem and Fermat's Little Theorem (CC-4)

Advice from faculty

The students of 2nd semester (Mathematics honours) are advised to attempt the enclosed assignment on Cosets and Lagrange's theorem. We have already conducted an extensive doubt clearing session on this topic via video conferencing on 15.05.2020. In case you have further queries while doing this assignment you can consult me over e-mail, WhatsApp or Google Classroom. You may submit the assignment in Google classroom or over mail (asutoshcollegemath@gmail.com) by 19.06.2020.

e-mail of faculty: prudra.math@gmail.com

Date: 12.06.2020

Assignment

Topics :- Cosets, Lagrange's theorem & FLT

1) Find all the distinct left cosets of $H = 6\mathbb{Z}$ in $(\mathbb{Z}, +)$

2) Let G be a group such that $|G| > 1$. Prove that G has only trivial subgroups iff $|G|$ is prime.

3) Let H be a subgroup of a group G. Denote by L_H the relation on G defined by

$$L_H = \{(a, b) \in G \times G \mid a^{-1}b \in H\}$$

Prove that,

i) L_H is an equivalence relation.

ii) Every equivalence class is a left coset of H in G.

iii) Every left coset of H is an equivalence class of the relation L_H.

4) Prove that every group of order 49 contains a subgroup of order 7.

5) Let G be a group such that $|G| < 320$. Suppose G has ~~order~~ subgroups of order 35 and 45. Find the order of G.

6) Let G be a group of order 15 and A and B subgroups of G of order 5 and 3 respectively. Show that $G = AB$.

7) Prove or disprove :-

i) Every proper subgroup of a group of order 25 is cyclic.

ii) Let $G = \langle a \rangle$ be a cyclic group of order 35. Then the index $[G : \langle a^7 \rangle] = 5$

iii) There may exist a subgroup of order sixteen in a group of order fifty.

<u>MCQ</u>

8) Suppose G has subgroups of order 45 and 75. If $|G| < 400$, then $|G|$ is

i) 90 ii) 150 iii) 225 iv) none of this.

9) The number of right cosets of the subgroup

$H = \{e, (12)(34), (14)(23), (13)(24)\}$

in S_4 is

i) 2 ii) 4 iii) 6 iv) 3

10) If $H \subset K$ are two normal subgroups of a group G, and if $[G : H] = 10$ and $[G : K] = 5$, then $[K : H]$ is

i) 5 ii) 2 iii) 10 iv) 50

11) The number of normal subgroup of order 4 in A_4 is

i) 0 ii) 2 iii) 4 iv) 1

12) If H and K be two subgroups of a group G and $o(G)/o(H) = o(G)/o(K)$, then

i) $H = K$ ii) $H \subset K$ iii) $H \supset K$ iv) $H \neq K$