WAVE-PARTICLE DUALITY: DE BROGLIE HYPOTHESIS

Although quantum theory has a brilliant success in the domain of interaction of light with matter such as Compton effect, Raman effect, Photo electric effect etc., it is very difficult to explain the phenomena of Interference, diffraction, Polarization etc. in the light of this theory. Thus electromagnetic theory (wave theory) has no place for photons (quanta) and the photon theory (Quantum theory) has no place for wave theory. Thus we are to adopt simultaneously two conflicting conceptions for the nature of light. A satisfactory theory must fuse these two conceptions together into one general coordinating principle. Thus the most significant aspect of Quantum theory is the wave-corpuscular dial nature of light.

Similarly, the French physicist L. de Broglie instituted the conception that matter, composed of discrete particles like molecules, atoms, photons, electrons, etc. must possess wave like properties under suitable circumstances. This means that matter, like radiation, has dual nature. According to wave mechanics, there is no ultimate particle of matter and everything is reducible to wave motion. Matter wave fives us the probability of finding the material particle at a given place. They guide the material particle in the same way as the electromagnetic wave guide the light quanta.

De Broglie developed his theory from the assumption that a frequency ϑ_0 can be associated with the rest mass energy of a particle according to Einstein's mass energy relation as

$$h\vartheta_0 = m_0c^2$$

Now, any quantity Ψ, which undergoes periodic vibrations and gives rise to matter waves, at any instant t_0 at a point (x_0, y_0, z_0) in a system at rest relative to the particle can be written as

$$\psi = \psi_0 Sin2\pi\vartheta_o t_0 \text{ -------------------------------------- (1)}$$

Where Ψ_0 is the amplitude of vibration at rest.

If now the **particle velocity** is v in x- direction, the vibration associated with the particle is equivalent to a progressive wave which can be represented by,

$$\psi = \psi_0 Sin\frac{2\pi}{T}\left(t - \frac{x}{u}\right) = \psi_0 Sin2\pi\vartheta(t - \frac{x}{u}) \text{ --------------------------------------- (2)}$$

Where T = period; ϑ = frequency; u = velocity of the **wave** in x- direction; x = phase difference.

According to the theory of relativity, t and t_0 are connected by the transformation equation

$$t_0 = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \text{ ------------------------------------- (3)}$$

Combining equation (1) and (3), we get

$$\psi = \psi_0 Sin2\pi\vartheta_0\left(\frac{t-\frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}\right) = \psi_0 Sin2\pi(\frac{\vartheta_0}{\sqrt{1-\frac{v^2}{c^2}}})(t-\frac{vx}{c^2}) \text{ ---------------------------- } (4)$$

Now, comparing equation (2) and (4) we have

$u = \frac{c^2}{v}$ and $\frac{1}{T} = \vartheta = \frac{\vartheta_0}{\sqrt{1-\frac{v^2}{c^2}}}$

Now, $\vartheta = \frac{\vartheta_0}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{m_0c^2/h}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}.\frac{c^2}{h} = \frac{mc^2}{h}$

Now the wave length of the matter wave is given by;

$\lambda = \frac{velocity}{frequency} = \frac{u}{\vartheta} = \frac{c^2/v}{mc^2/h} = \frac{h}{mv} = \frac{h}{p}$; where p = mv = momentum of the particle

So, $\boldsymbol{\lambda = \frac{h}{p}}$ **----------** This is known as **De Broglie wavelength** and **De Broglie equation**.

If an electron of mass "m" and charge "e" e.s.u. moves under potential difference of V volts, then

$$\frac{1}{2}mv^2 = \frac{eV}{300} \Rightarrow m^2v^2 = \frac{2meV}{300} = \frac{meV}{150} \Rightarrow p = mv = \sqrt{\frac{meV}{150}}$$

$$So, \lambda = h\sqrt{\frac{150}{meV}}$$

Using the values of m = 9.1×10^{-28} gm; e = 4.8×10^{-10} e.s.u.; h = 6.62×10^{-27} erg-sec

$$\boldsymbol{\lambda = \sqrt{\frac{150}{V}} \times 10^{-8}\, cm} = \sqrt{\frac{150}{V}}\ \text{Å} \approx \frac{12.27}{\sqrt{V}}\ \text{Å} \text{ ------------------ } (5)$$

Applying relativistic correction for electrons moving with a velocity comparable to that of light, this equation reduces to;

$\lambda = \frac{12.27}{\sqrt{V}}(1+\frac{\alpha}{2})^{-\frac{1}{2}}$; where $\alpha = eV/(300m_0c^2)$ ------------------ (6)

Equation (5) and (6) gives the quantitative measure of the wavelength of the matter waves.