

LASER (Light Amplification by Stimulated Emission of Radiation)

Properties of LASER light:- It is a source which emits an intense perfectly monochromatic, directional and highly coherent beam of light.

A LASER is fundamentally a light source having

some properties:-

- i). Directionality:- The divergence of a laser beam is limited to less than 10^{-5} radians by diffraction. This property finds application in surveying, remote sensing, lidar etc.
- ii). High Power:- The power level of continuous wave laser is $\sim 10^5$ watt, whereas for pulsed laser it is ~ 50000 J. So it is used in welding, cutting etc.
- iii). Tight focussing:- Due to high degree of directionality laser beam can be focussed within a small area $\sim$ a few $(\mu\text{m})^2$ leading to large amount of energy ($\sim 12 \times 10^5$ watt/ m^2). That is why it is used in surgery, compact disc, material processing etc.
- iv). Spectral purity:- The spectral width ($\Delta\lambda$) of laser beam is very small i.e. $\Delta\lambda \sim 10^{-6}$ Å, which can be further reduced by adopting suitable technique. This property is successfully applied in spectroscopy, holography, optical communication etc.
- v). High degree of coherence and continuous power output from microwatt to kilowatt.

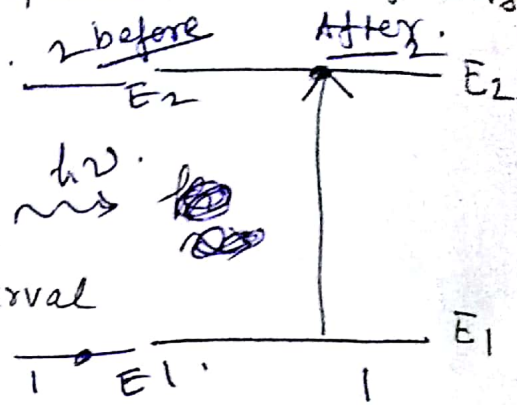
BASIC PRINCIPLE OF LASER:- According to N. Bohr, the atoms are characterized by discrete energy levels. Einstein proposed that such atoms can interact with incident e.m. waves in the following three ways:-

- 1) Stimulated absorption
- 2) Spontaneous emission
- 3) Stimulated emission.

1) Stimulated absorption:- Let an atom have two energy states E_1 and E_2 ($E_2 > E_1$). When a photon of ~~energy~~ freq. ν incident on

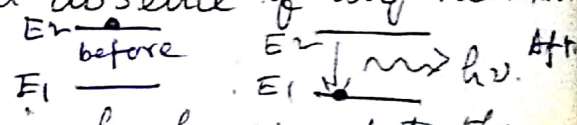
on such an atom so that $\nu = \frac{E_2 - E_1}{h}$, then the atom is pumped to the excited state E_2 . The rate of stimulated absorption (or, simply absorption) depends on two factors

- 1) The intensity of incident radiation.
 - 2) The number of atoms in state E_1 .
- By this process the density of photons in the incident beam decreases. The atom in state E_2 can exist for a small interval of time $\tau \approx 10^{-8} \text{ sec}$, the life time of the excited state.



2) Spontaneous Emission:— The life time of the excited state being very small ($\approx 10^{-8} \text{ s}$), an atom in energy state E_2 transits to the state E_1 , giving rise to the emission of a photon of freq. $\nu = \frac{E_2 - E_1}{h}$ where $h = 6.626 \times 10^{-34} \text{ J-s}$.

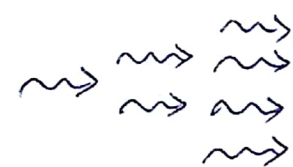
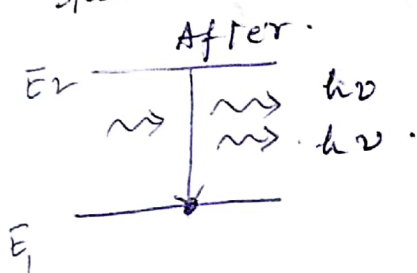
- Since the process can take place in absence of any radiation it is called spontaneous emission.
- The rate of spontaneous transition is proportional to the no of atoms in the excited state E_2 and independent of exciting radiation.
- It is ~~independent of the ex~~ random in nature having no definite phase relationship among the emitted waves of various wavelength. The radiation is thus incoherent.



3) Stimulated emission:— Einstein proposed that an atom in the excited state E_2 can transit to the ground state E_1 when they are stimulated (or induced externally) by same energy as that of de-excitation.

- The rate of stimulated emission depends on the number of atoms in the excited state and intensity of external radiation.
- As the freq, phase and state of polarisation of stimulated emission are the same as incident photon, it is highly coherent.
- The two photons emitted in this process can interact with

two other atoms in the excited state E_2 and can again induce stimulated emission, giving four photons. In this way a chain process is continuously increasing the density of photon and the light amplification (laser action) occurs. For this chain process to sustain the condition is: the number of atom in excited state must be greater than ^{that} in the ground state. i.e. $N_2 > N_1$. The highly populated energy state is called the population inversion. BEFORE E_2



generation of photons by chain reaction.

Spontaneous Emission.

- 1) This process takes place in absence of any radiation.
- 2) The rate of emission is proportional to the number of atoms in the excited state and independent of the incident intensity.
- 3) It is random in character and a random mixture of quanta having various wavelengths.
- 4) The waves coincide ~~not~~ neither in freq nor in phase and as such the radiation is incoherent and has a broad spectrum.
- 5) It has no relevance with light amplification.

Stimulated Emission.

- 1) This process is triggered by an incident radiation of proper freq.
- 2) The rate of transition is proportional to both the number of atoms in the excited state as well as the incident intensity.
- 3) It is highly directional and all the quanta have the same wavelength.
- 4) The stimulated photon is exactly of the same freq, phase, direction and state of polarisation as the incident photon. So it is coherent.
- 5) It can develop a chain reaction leading to light amplification and high intensity ~~and~~ coherent radiation.

LASER and Ordinary Light -

LASER

- 1) It is highly directional, the divergence being limited to $\sim 10^{-5}$ radians.
- 2) High peak power and large energy per pulse in pulsed output laser.
- 3) LASER is highly monochromatic.
- 4) It can be focussed in an area $\sim \lambda^2$.
- 5) Laser is perfectly coherent, since the phase and freq. of all the emitted waves are the same.

Ordinary Light -

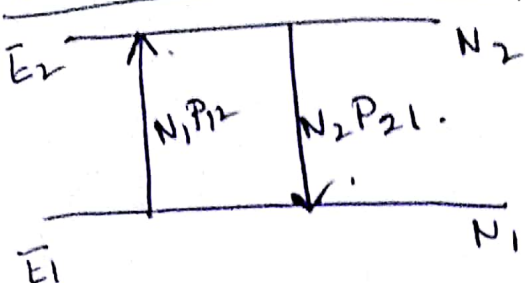
- 1) It is random in character, the divergence being larger than laser beam.
- 2) The power of ordinary light is much greater than laser.
- 3) Ordinary monochromatic light has a wider range of freq. in comparison to laser.
- (4) It cannot be focussed in an area $\sim \lambda^2$.
- (5) It is incoherent, there being no const phase and freq. relationship.

The Einstein Co-efficients

Let us consider a system having two energy states E_1 and E_2 ($E_2 > E_1$) having number of atoms N_1 and N_2 per unit volume respectively at temp T . Then according to Maxwell-Boltzmann distribution law, the number of atoms in states E_1 and E_2 are $N_1 = N_0 e^{-E_1/kT}$ and $N_2 = N_0 e^{-E_2/kT}$

$\therefore \frac{N_2}{N_1} = e^{-(E_2 - E_1)/kT}$ where k is Boltzmann const and N_0 is the number of atoms in the ground state E_0 . From ① $\rightarrow N_2 < N_1$ as $E_2 > E_1$.

INDUCED ABSORPTION:-



An atom in the lower energy level can absorb radiation and get excited to the level E_2 . This excitation process can occur only in the presence of radiation.

Such a process is called absorption. The rate of absorption depends on the density of radiation at the particular freq. corresponding to the energy separation of the two levels. Thus, if $\nu = \frac{E_2 - E_1}{h}$ then the absorption process depends on the energy density of radiation at freq ν . This energy density is denoted by $u(\nu)$ and is defined such that $u(\nu)d\nu$ represents the radiation energy per unit volume within the freq interval ν and $\nu + d\nu$. The rate of absorption is proportional to N_1 and also on $u(\nu)$.

$$\therefore \frac{dN_{12}}{dt} \propto N_1 u(\nu)$$
$$\text{or, } \frac{dN_{12}}{dt} = B_{12} N_1 u(\nu)$$

Where B_{12} is the co-eff of proportionality and is a characteristics of the energy level and called

Einstein Co-eff of induced absorption.

SPONTANEOUS EMISSION:-

Let us now consider the reverse process i.e. the emission of radiation at a freq ν when the atom de-excites from level E_2 to E_1 .

Now, an atom can make a radiative transition to a lower energy level either through spontaneous emission or through stimulated emission. In spontaneous emission, the rate of emission/the probability of per unit time of an atom to make downward transition is independent of energy density of the radiation and depends only on the number of atoms on the upper levels involved in the transition.

$$\text{Then } \frac{dN_{21}}{dt} \propto N_2 \quad \text{or, } \frac{dN_{21}}{dt} = A_{21} N_2$$

where A_{21} is Einstein Co-eff of spontaneous emission.

LASER

⇒ Stimulated emission:- The rate of transition to lower energy level is directly proportional to the number of atoms in the upper energy level as well as to the energy density of the radiation at the freq ν .

$$\text{Thus the rate of stimulated emissions} = \frac{dN_{21}}{dt} \propto N_2 u(\nu) = B_{21} N_2 u(\nu)$$

where $B_{21} \rightarrow$ Einstein Co-eff for stimulated emission.

At thermal equilibrium, the number of upward transitions must be equal to the number of downward transitions.

$$\therefore N_1 B_{12} u(\nu) = N_2 A_{21} + N_2 B_{21} u(\nu)$$

$$\text{or, } u(\nu) = \frac{A_{21} N_2}{N_1 B_{12} - N_2 B_{21}} \rightarrow (1)$$

Again from Boltzmann law, $\frac{N_1}{N_2} = \exp\left(\frac{E_2 - E_1}{kT}\right) = \exp\left(\frac{h\nu}{kT}\right)$ where k is Boltzmann const. $\rightarrow (2)$

$\therefore$ eqn (1) becomes,

$$u(\nu) = \frac{A_{21}}{\frac{N_1}{N_2} B_{12} - B_{21}} = \frac{A_{21}}{B_{12} \exp\left(\frac{h\nu}{kT}\right) - B_{21}} \rightarrow (3)$$

According to Planck's law,

$$u(\nu) = \frac{8\pi h \nu^3}{c^3} \frac{1}{\exp\left(\frac{h\nu}{kT}\right) - 1} \rightarrow (4)$$

So comparing (3) and (4), $B_{12} = B_{21} = B$ (say)

$$\text{and } \frac{A_{21}}{B_{21}} = \frac{8\pi h \nu^3}{c^3} = \frac{8\pi}{c^3 h^2} (E_2 - E_1)^3 \rightarrow (5)$$

Again from (3); at thermal equilibrium, the ratio of the number of spontaneous to stimulated emission is

$$\frac{A_{21}}{B_{21} u(\nu)} = \exp\left(\frac{h\nu}{kT}\right) - 1$$

for normal optical source $T \sim 10^3$ °K. with
 $\nu = 3 \times 10^{15} \text{ sec}^{-1}$ corresponding $\lambda = 6000$ Å.

$$\frac{h\nu}{kT} \approx 23.$$

giving $\frac{A_{21}}{B_{21} u(\nu)} \approx 10^{10}$.

Thus at optical fr. the emission is predominantly due to spontaneous transition and hence the emission from

usual light source is incoherent.

when $h\nu \ll kT$; stimulated emission is much greater than spontaneous emissions, when $h\nu \gg kT$, the reverse is true.

Now the ratio of stimulated emission to the rate of spontaneous emission is given by,

$$P = \frac{B_{21} N_2 E(\nu)}{A_{21} N_2} = \frac{B_{21} \times 8\pi h\nu^3}{A_{21} c^3} \frac{1}{e^{h\nu/kT} - 1} \rightarrow (6)$$

$$\text{or, } P = \frac{1}{e^{h\nu/kT} - 1} \rightarrow (7)$$

from (6). if (a) $h\nu \ll kT$ i.e. in microwave region at room temp. probability of stimulated emission is greater than that of spontaneous emission. if incident radiation is in microwave region. ($\lambda \sim 10 \text{ cm}$, $\nu = 3 \times 10^9 \text{ s}^{-1}$, $T = 300 \text{ K}$) (MASER)

(b) In the visible region, $T \sim 10^3 \text{ K}$, $\lambda \sim 6500 \text{ Å}$ $\therefore P \sim 10^{-58}$
 Hence stimulated emission is almost absent, spontaneous emission forms the major component of emission.

(c) Condition for higher stimulated emission than absorption

at thermal equilibrium. $\frac{\text{Stimulated emission rate}}{\text{Stimulated absorption rate}} = \frac{N_2 B_{21} E(\nu)}{N_1 B_{12} E(\nu)}$

$$\text{or, } P_1 = \frac{N_2}{N_1} \quad [\because B_{12} = B_{21}]$$

$\therefore$ at thermal equilibrium, $N_1 \gg N_2$, $P_1 \ll 1$. i.e. the stimulated absorption rate dominates the stimulated emission rate. This therefore can't be used for LASER action so the light is incoherent.

to have light amplification. $N_2 \gg N_1$, i.e. the state E_2 must be more populated than state E_1 . Under this condition stimulated emission rate will be greater than the corresponding stimulated absorption rate when the photon density is increasing and light amplification by stimulated emission of radiation (LASER) occurs.

The state for which $N_2 \gg N_1$ is called population inversion, when the number of atoms in the higher energy state (E_2) is very much greater than that of lower energy state (E_1).

Hence population inversion is an essential condition for laser action to take place.

(d) The population inversion is realized if there be a metastable state (E_0) such that $E_1 < E_0 < E_2$ having life time considerably large ($\sim 10^{-3}$).

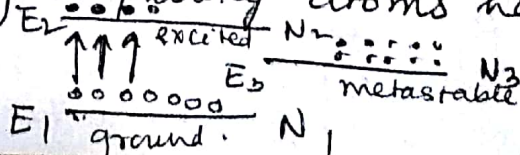
(e) $h\nu \gg kT$, there does not exist any atom in higher energy state, so for low temp. higher energy state is almost empty.

(f) So as temp increase N_2 approaches but not exceed it [at $h\nu = kT$, $\frac{N_2}{N_1} = \frac{1}{2}$].

By pump, ~~we~~ artificially more atoms are excited to higher energy state to achieve population inversion.

Metastable state:— For population inversion to be achieved, the life time of excited state must be 10^{-6} to 10^{-3} s.

(~~which is~~ 10^{-8} s for spontaneous emission). ~~and~~ ~~which is~~ 10^3 to 10^6 time the life time of ordinary energy level. This energy state is called metastable state, which allows accumulation of large no. of excited atoms. Metastable state in a crystal can be obtained by doping impurity atoms having energy level in the forbidden gap.



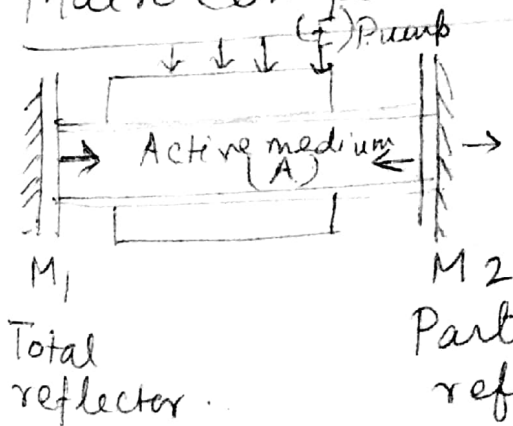
Achievement of Population Inversion: Pumping

For any laser device to operate, the achievement of population inversion in the active medium is necessary. That is, a large number of atoms of the medium should be excited to higher energy states. The process by which atoms are raised from a lower to a higher energy state is called 'pumping' of atoms.

The pumping process in laser involves a number of energy levels with complex excitation processes. Different methods of atomic excitation are adopted in different types of lasers. Some of the methods are:-

- i) excitation by a strong source of light, say flash lamp or arc lamp (optical pumping).
- ii) excitation by electron impact (electrical pumping)
- iii) excitation by chemical reaction (chemical pumping)
- iv) excitation by supersonic gas expansion (gas-dynamic pumping).

Main Components of LASER:-



- 1) Active medium:- An inversely populated medium in which light gets amplified is called an active medium. It may be solid, liquid, gas or semiconductor junction having the capacity to sustain stimulated emission.

(2) An external source of excitation: - The pumping source (E) supplies pumping energy to create population inversion.

(3) Optical resonator: The optical resonator may be a pair of plane parallel mirrors M_1 and M_2 . M_1 is total reflector, and M_2 is partial. M_2 allows a part of laser beam L to pass out.

Action: - The medium is irradiated with pump energy E. The excited atoms release this energy by stimulated emission. A photon released by one excited atom encounters a second excited atom in its path and stimulates it to release a second ~~atom~~ photon. These ^{two} ~~intensity~~ coherent photons make the intensity ^L of the beam twice. The intensity of the light beam increases rapidly, more stimulated photons as it passes through the medium. The beam being reflected by more stimulated photons as it passes through the medium. The beam being reflected by M_1 undergoes another passage through the excited medium and gets further amplified. Reaching M_2 , a part L of the amplified beam passes out as a laser beam.

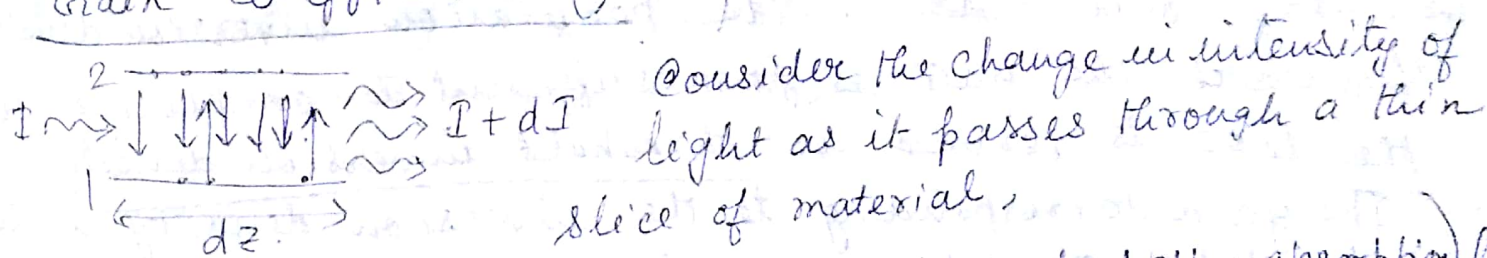
Theory of Amplification of light.

Let a beam of light having intensity I_0 passes through a medium of length x . The intensity of the beam after leaving the medium by Beer's Law,
 $I(x) = I_0 e^{-\alpha x}$ where $\alpha \rightarrow$ absorption co-eff of medium.

Due to stimulated absorption by the medium, the intensity of light beam will decrease.

But, if the medium is active, i.e. inversely populated, then the stimulated emission will predominate over stimulated absorption. As a result, the light beam undergoes amplification instead of attenuation during passing through medium. $I(x) = I_0 e^{\beta x}$ where β is gain or amplification co-efficient.

Gain Co-efficient (β) / Optical gain.



$$dI = \left(\text{rate of stim. emission per vol} - \text{rate of stim. absorption per vol} \right) \times dz$$

Where spontaneous emission has been neglected.

Since $B_{21} = B_{12}$

$$\therefore \frac{dI}{dz} = (N_2 B_{21} - N_1 B_{12}) u(\nu) h\nu$$

$$= (N_2 B_{21} - N_1 B_{12}) \frac{I}{c} h\nu$$

$$= (N_2 - N_1) B_{21} \frac{I}{c} h\nu \rightarrow (1)$$

$$\therefore I = I_0 e^{\beta z}$$

$$\therefore \frac{dI}{dz} = I_0 \beta e^{\beta z} = \beta I \rightarrow (2)$$

so comparing (1) and (2).

$$\beta = \frac{(N_2 - N_1) B_{21} h\nu}{c} = (N_2 - N_1) \sigma$$

where σ is stimulated emission cross section.

light amplification will occur if β is positive.
i.e. $N_2 > N_1$, i.e. when population inversion is achieved.
Under such condition the light intensity of photon increases while passing through the medium.

Threshold condition for light amplification.

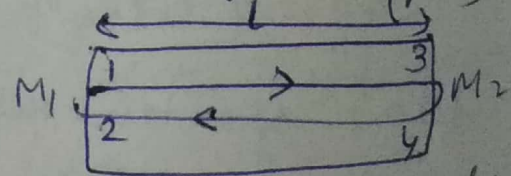
During light amplification some photons are lost due to absorption and scattering. If the population density of the medium is sufficiently large to compensate the loss, then only lasing action will occur. The population inversion density for which the gain is just sufficient to compensate for the loss is termed as threshold inversion density.

The gain corresponding to the inversion density under threshold condition is called threshold gain (β_{th}).

Calculation of β_{th} :-

Let two plane mirrors M_1 and M_2

with reflectivity R_1, R_2 be placed at the two ends of the active medium of length l .



Let I_0 = intensity of the incident beam (1) on M_1 .

Then the intensity of the reflected beam (2) at $M_1 = I_0 R_1$.

β = gain of the medium, then the intensity of beam 3

is $I_0 R_1 e^{\beta l}$ and that of beam 4 reflected at M_2 is $I_0 R_1 R_2 e^{2\beta l}$.

$\therefore$ gain of the medium (G)

= intensity of the beam at the end of one round trip

Intensity of the beam at the start of round trip.

$$= \frac{I_0 R_1 R_2 e^{2\beta l}}{I_0}$$

$$\text{or, } G = R_1 R_2 e^{2\beta l}$$

At threshold $G = I_0$, as the intensity of the beam does not change,

$$\text{or, } I_0 R_1 R_2 e^{2\beta_{th} l} = I_0 \quad [\text{where } \beta = \beta_{th} \text{ at threshold}]$$

$$\text{or, } e^{2\beta_{th} l} = \frac{1}{R_1 R_2}$$

$$2\beta_{th} l = \ln\left(\frac{1}{R_1 R_2}\right), \quad \text{or, } \beta_{th} = \frac{1}{2l} \ln\left(\frac{1}{R_1 R_2}\right)$$

If the value of β_{th} satisfies the above relation then continuous light amplification will occur.

Rate equation for a two level system.

Consider a two level system with upper level u and ground level g .

In order that, laser transition may occur, we need population inversion.

If w_p is the pumping induced transition rate for $g \rightarrow u$ or, $u \rightarrow g$ and τ is ~~not~~ natural life time of atoms in upper level, the rate eqn. for the two levels may be written as,

$$\frac{dN_u}{dt} = - \frac{dN_g}{dt} = w_p (N_p - N_u) - \frac{N_u}{\tau}$$

Where $N_p + N_u = N = \text{const}$

In steady state, the time derivative vanishes

$$\frac{N_u}{\tau} = w_p (N_p - N_u)$$

$$\text{or, } \frac{N_u}{\tau} = w_p N_p - w_p N_u$$

$$\text{or, } N_u \left(\frac{1}{\tau} + w_p \right) = w_p N_p$$

$$\text{or, } N_u (1 + w_p \tau) = \tau w_p N_p$$

$$\therefore N_u = \frac{N_p \tau w_p}{1 + w_p \tau}$$
$$N_p = (N - N_u)$$
$$= \frac{(N - N_u) \tau w_p}{1 + w_p \tau}$$

$$\text{or, } N_u (1 + w_p \tau) = N \tau w_p$$

$$\therefore N_u = \frac{N \tau w_p}{(1 + w_p \tau)^2} = \frac{N \tau w_p}{1 + 2w_p \tau}$$

for population inversion to take place, we must have $N_u > N/2$.

However one sees that, as the intensity is increased the population in the upper level at best approaches this number as its maximum. Thus population inversion is not possible in a two level system.

Three Level system:-

Fig (1) shows the Boltzmann distribution of energy. States with

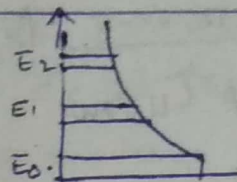


Fig (1)

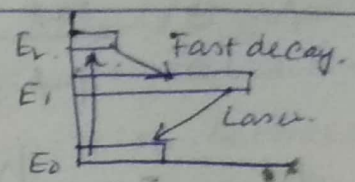
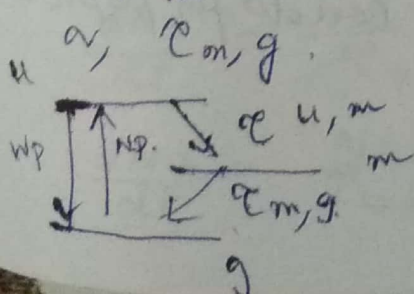


Fig (2)

no pumping. If the atoms in E_0 are intensely pumped, a good number of them are excited to E_2 through stimulated absorption. If the energy level E_2 is short lived, then the atoms decay fast to level E_1 . If E_1 is metastable state as in Fig 2, then the atoms are accumulated at E_1 . Thus with intense pumping from E_0 to E_2 and then rapid decay to E_1 , it is possible to achieve non-equilibrium distribution of atoms. Then E_1 is more populated than E_0 , when LASER transition takes place between E_1 and E_0 .

It is not possible to continuously maintain E_1 more populated than E_0 due to depopulation of E_1 . Such a system (e.g Ruby Laser) works in pulsed mode only.

Rate eqn.:- As the metastable state is long lived. it is assumed that $\tau_{u,m}$ is much smaller than $\tau_{u,g}$.



Rate eqn for level u ,

$$\frac{dN_u}{dt} = W_p(N_g - N_u) - \frac{N_u}{\tau_{u,m}}$$

Rate eqn for metastable state,

$$\frac{dN_u}{dt} = \frac{N_u}{\tau_{u,m}} - \frac{N_m}{\tau_{m,g}}$$

$$\therefore \frac{dN_g}{dt} = -\frac{d}{dt}(N_u + N_m) = -W_p(N_g - N_u) + \frac{N_m}{\tau_{m,g}}$$

In steady state, time derivatives are zero,

$$N_u = \frac{W_p \tau_{u,m}}{1 + W_p \tau_{u,m}} N_g$$

$$N_m = \frac{W_p \tau_{m,g}}{W_p \tau_{u,m} + 1} N_g$$

$\therefore$ The eqn for population inversion becomes

$$\frac{\Delta N}{N} = \frac{N_m - N_u}{N} = \frac{W_p \tau_{m,g} (1 - \beta) - 1}{W_p \tau_{m,g} (1 + 2\beta) + 1} \rightarrow (A)$$

$$\text{Where } \beta = \frac{N_u}{N_m} = \frac{\tau_{u,m}}{\tau_{m,g}} \rightarrow (B)$$

for population inversion to occur, the numerator of eqn (A) must be (+ve)

$$\therefore W_p \tau_{m,g} (1 - \beta) > 1$$

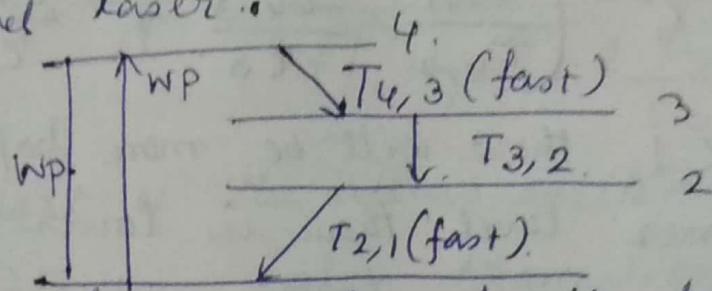
$$\text{i.e. } W_p \tau_{m,g} > \frac{1}{1 - \beta} \rightarrow (C)$$

$$\text{from (B) } \beta < 1 \therefore \tau_{u,m} < \tau_{m,g}$$

$\therefore$ $\boxed{W_p \tau_{m,g} > 1}$ is sufficient condⁿ for population inversion.

Four Level System:- In four level system, atoms are lifted from ground state (E_0) to the highest state E_3 by pumping. From E_3 state, atoms decay to E_2 state, (metastable state) and population in this state rapidly increases. A population inversion of $E_2 \rightarrow E_1$ transition can be achieved if the life time of $E_3 \rightarrow E_2$ transition is shorter compared to $E_2 \rightarrow E_1$ transition with moderate pumping. Since here E_1 is the lower laser level, it is comparatively easier to maintain inversion between E_2 and E_1 continuously with moderate pumping and get continuous laser beam. For this $E_1 \rightarrow E_0$ transition must be very fast. If the transition is slower then the laser source will give pulsed beam only. He-Ne laser is a good example of four level laser.

Rate equation:-



Let τ be the lifetime of the level 4 for decay into any of the lower states. As the decay constants are additive,

$$\frac{1}{\tau_4} = \frac{1}{\tau_{4,3}} + \frac{1}{\tau_{4,2}} + \frac{1}{\tau_{4,1}}$$

Similarly, $\frac{1}{\tau_3} = \frac{1}{\tau_{3,2}} + \frac{1}{\tau_{3,1}}$

$\therefore$ Rate eqns are,

$$\frac{dN_4}{dt} = W_p (N_1 - N_4) - \frac{N_4}{\tau_4} \rightarrow (1)$$

$$\frac{dN_3}{dt} = \frac{N_4}{\tau_{4,3}} - \frac{N_3}{\tau_3} \rightarrow (2)$$

$$\frac{dN_2}{dt} = \frac{N_4}{\tau_{4,2}} + \frac{N_3}{\tau_{3,2}} - \frac{N_2}{\tau_{2,1}} \rightarrow (3)$$

$$N = N_1 + N_2 + N_3 + N_4$$

In steady state all the time derivative will be zero,

from (1) $N_4 = \frac{W_p \tau_4}{1 + W_p \tau_4} N_1 \rightarrow (4)$

from (2) $N_3 = \frac{\tau_3}{\tau_{4,3}} N_4 \rightarrow (5)$

from (3)
$$N_2 = \frac{\tau_{2,1}}{\tau_{4,2}} N_4 + \frac{\tau_{2,1}}{\tau_{3,2}} N_3$$

$$= \left(\frac{\tau_{2,1}}{\tau_{4,2}} \cdot \frac{\tau_{4,3}}{\tau_3} + \frac{\tau_{2,1}}{\tau_{3,2}} \right) N_3$$

$$= \beta N_3 \rightarrow (6)$$

where $\beta = \left(\frac{\tau_{2,1}}{\tau_{4,2}} \cdot \frac{\tau_{4,3}}{\tau_3} + \frac{\tau_{2,1}}{\tau_{3,2}} \right)$

If $\beta < 1$, there will be more population in the upper laser level than in lower leading to population inversion.

Simplifying

$$N = \frac{1 + 2W_p \tau_4 + (1 + \beta) W_p \frac{\tau_4 \tau_3}{\tau_{4,3}}}{1 + W_p \tau_4} N_1$$

and

population difference $N_3 - N_2 = \Delta N$

$$= (1 - \beta) \frac{\tau_3}{\tau_{4,3}} \frac{W_p \tau_4}{1 + W_p \tau_4} N_1$$

Combining

$$\frac{\Delta N}{N} = \frac{(1 - \beta) W_p \frac{\tau_3 \tau_4}{\tau_{4,3}}}{1 + \left[1 + \beta + 2 \frac{\tau_{4,3}}{\tau_3} \right] W_p \frac{\tau_4 \tau_3}{\tau_{4,3}}}$$

define η = quantum efficiency as the fraction of atoms excited from ground state (1), which ultimately results stimulated emission. It is the product of fraction which arrives at level (3) and the fraction of atoms in upper laser level which make radiative transition to lower laser level (2).

Now fraction of atoms in level 4 which arrive at level 3 is given by the ratio $\tau_4/\tau_{4,3}$ and the fraction of atoms in level 3 which radiatively make a transition to level 2 is $\tau_3/\tau_{3,2}^{\text{rad}}$.

$$\eta = \frac{\tau_4}{\tau_{4,3}} \cdot \frac{\tau_3}{\tau_{3,2}^{\text{rad}}}$$

Substituting this;
$$\frac{\Delta N}{N} = \frac{(1-\beta)W_p \eta \tau_{3,2}^{\text{rad}}}{1 + \left[(1+\beta) + 2 \frac{\tau_{4,3}}{\tau_4} \right] W_p \eta \tau_{3,2}^{\text{rad}}}$$

To ensure that most of the atoms excited by pumping participate in laser transition, the life time in level 3 must be the longest.

Using $\tau_{4,3} \ll \tau_3$, it is ignored in the denominator of the above relation of η .

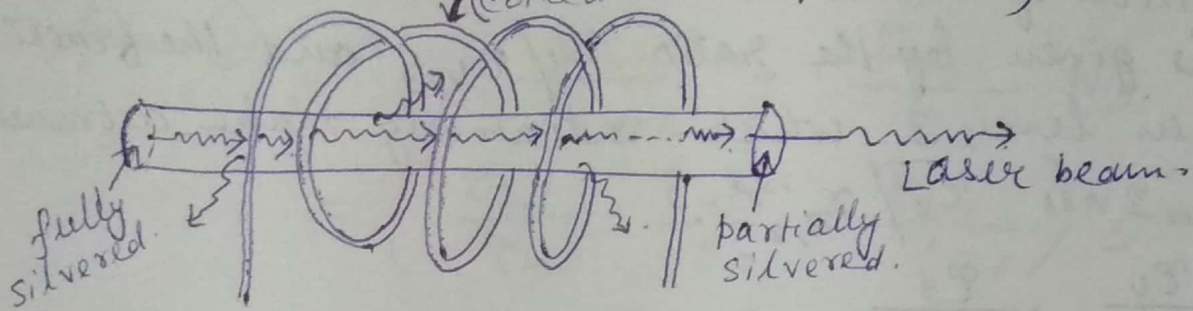
Further, $\tau_{3,2} \gg \tau_{2,1}$. Using these, one can see that β is small and approaches zero.

Thus $\Delta N/N$ remains positive and population inversion occurs even for very small pumping power.

For $\beta \rightarrow 0$, the expression for $\Delta N/N$ is given by,

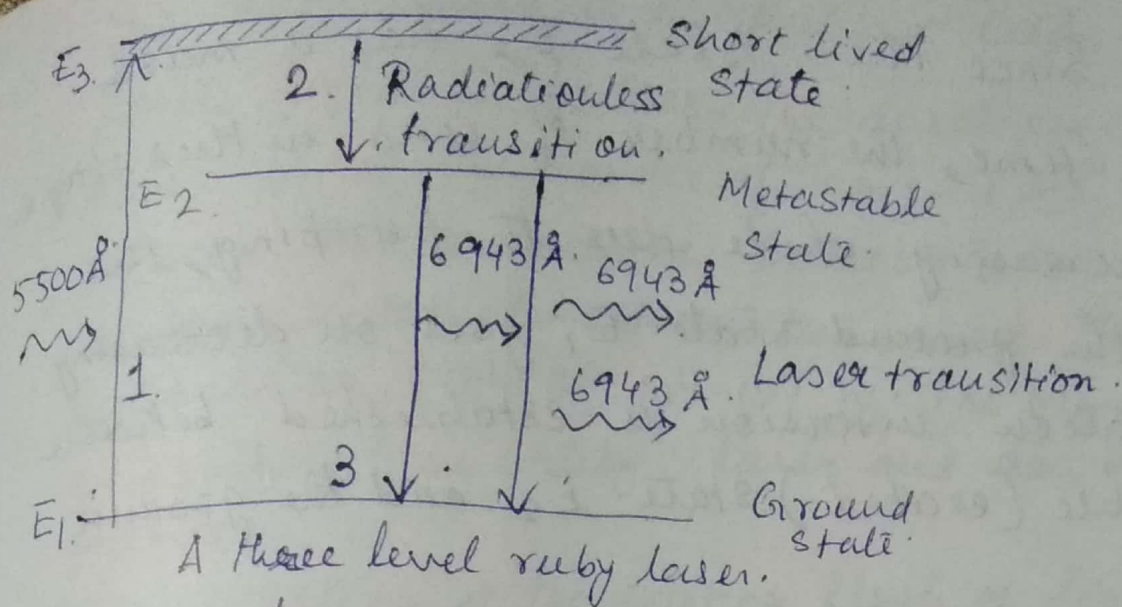
$$\frac{\Delta N}{N} = \frac{W_p \eta \tau_{3,2}^{\text{rad}}}{1 + W_p \eta \tau_{3,2}^{\text{rad}}}$$

The Ruby Laser:- The ruby laser was the first laser developed in 1960. It is a solid state laser. It consists of a pink ruby cylindrical rod whose ends are optically flat and parallel. One end is fully silvered and the other is partially silvered, and the other is partially silvered, (coiled xenon flash lamp)



The two ends thus form a resonant cavity. The ruby rod, is placed inside a coiled xenon flash lamp. The flash lamp is connected to a capacitor, (not shown) which discharges a few thousand joules of energy in a few milliseconds, resulting in a power output of a few megawatts from the flash lamp.

Working:- The ~~rod~~ ruby rod is a crystal of aluminium oxide (Al_2O_3) doped with 0.05% chromium oxide (Cr_2O_3), so that some of the Al^{++} ions are replaced by Cr^{+++} ions. These "impurity" chromium ions give pink colour to the ruby and give rise to laser action.



↑ Energy level diagram of chromium ion. It consists of an upper short-lived energy level (energy band) E_3 about its ground state energy level E_1 , the energy difference $E_3 - E_1$ corresponding to a wavelength of about 5500 Å . There is an intermediate excited state level E_2 which is metastable having a lifetime of $3 \times 10^{-3} \text{ s}$ (about 10^5 times greater than the lifetime of E_3 which is $\sim 10^{-8} \text{ sec}$).

Normally most of the Chromium ions are in ground state E_1 . When a flash of light (which lasts only for about a millisecond) falls upon the ruby rod, the 5000 Å radiation photons are absorbed by the chromium ions which are 'pumped' or 'raised' to the excited state E_3 . The transition 1 is the (optical) pumping transition. The excited ions rapidly give up, by collision, part of their energy to the crystal lattice and decay to the 'metastable' state E_2 . The corresponding transition 2 is thus a radiationless

transition. Since the state E_2 has a much longer life-time, the number of ions in this stage goes on increasing while due to pumping, the number in the ground state E_1 goes on decreasing. Thus population inversion is established between the metastable (excited) state E_2 and the ground state E_1 .

When an (excited) ion passes spontaneously from the metastable state to the ground state (transition 3), it emits a photon of wavelength $6943 \text{ \AA}$. This photon travels through the ruby rod and if it is moving parallel to the axis of the crystal, is reflected back and forth by the silvered ends until it stimulates an excited ion and causes it to emit a fresh photon in phase with the stimulating photon.

This stimulated transition 4 is the laser transition. (The photons emitted spontaneously which do not move axially escape through the sides of the crystal). The process is repeated again and again because the photons repeatedly move along the crystal, being reflected from its ends. The photons thus multiply. When the photon beam becomes sufficiently intense, part of it emerges through the partially silvered end of crystal.

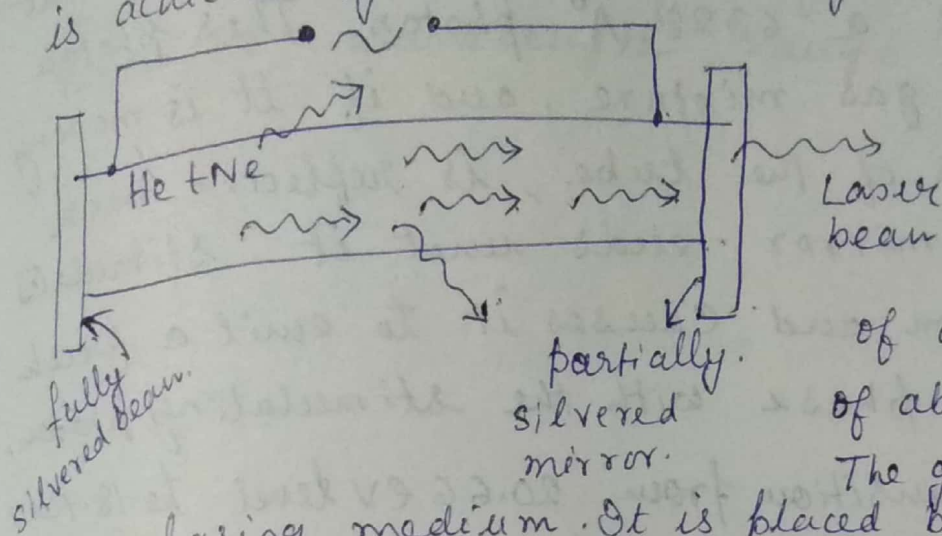
The ruby laser is a 'pulsed' laser, i.e. it emits laser light in pulses. The duration of an individual pulse is of the order of $0.1 - 1 \mu s$, the time interval between two successive pulses is about $1 - 10 \mu s$. ~~The time interval between two successive~~ The power of each pulse is as high as $10^4 - 10^5$ W. The phenomenon is known as 'spiking' in ruby laser and can be explained as follows:

The duration of the exciting flash of light is of the order of a millisecond, but it is intense enough to build up population inversion very rapidly. As soon as a larger population is produced in the upper level, the laser action starts producing a pulse. This depletes the upper level population more rapidly than it is restored by the flash light. The laser action then stops for a few micro seconds. In the meantime, the flash lamp again builds up population inversion producing another pulse. This sequence is repeated. Hence a series of pulses is produced until the intensity of the flash light falls to such a level that it can no longer rebuild the required population inversion.

Drawback:— There is a drawback in the three level laser such as ruby. The laser requires very ~~much~~ high pumping power, because the laser transition terminates at the ground state, and more than one third of the ground state atoms must be pumped to the higher state to achieve population inversion.

Helium - Neon Laser:-

The Helium-Neon Laser was the first gas Laser to be operated successfully. It was fabricated by Tavan in 1961. It is a four level laser in which population inversion is achieved by electric discharge.

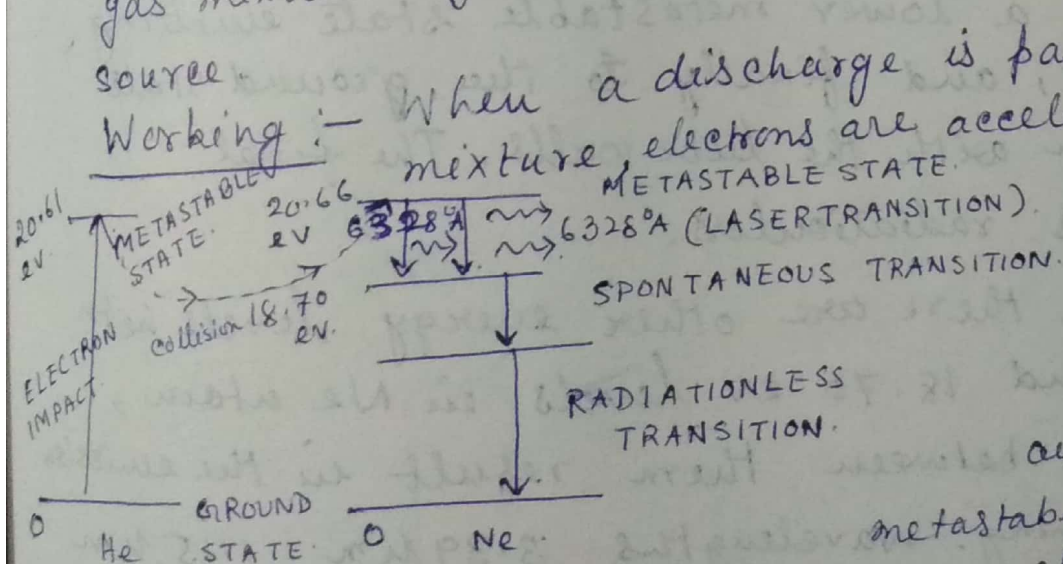


The He-Ne laser consists of a long and narrow discharge tube filled with a mixture of He and Ne in a ratio

of about 10:1 at a pressure of about 1 mm of mercury.

The gas mixture (He + Ne) forms the lasing medium. It is placed between a pair of optically plane and parallel mirrors which forms a resonant cavity. One of the mirrors is fully silvered and the other is partially silvered. The spacing of the mirrors is equal to an integral number of half-wave lengths of the laser light. An electric discharge may be produced in the gas mixture by electrodes connected to a high freq. electric source.

Working:-



When a discharge is passed through a gas mixture, electrons are accelerated down the tube. These accelerate electrons collide with and pump (excite) the He

and Ne atoms to metastable states 20.61 eV and 20.66 eV respectively above their ground state.

Some of the excited He atoms transfer their energy to ground state Ne atoms by collision with 0.05 eV of additional energy being provided by the kinetic energy of atoms. Thus He atoms help in achieving a population

inversion in the Ne atoms.

When an excited Ne atom passes spontaneously from the metastable state at 20.66 eV to the state at 18.70 eV, it emits a 6328-Å photon. This photon travels through the gas mixture, and if it is moving parallel to the axis of the tube, is reflected back and forth by the mirror-ends until it stimulates an excited Ne atom and causes it to emit a fresh 6328 Å photon in phase with the stimulating photon. This stimulated transition from 20.66 eV level to 18.70 eV level is the laser transition. This process is continued and a beam of coherent radiation builds up in the tube. When this red-light beam becomes sufficiently intense, a portion of it escapes through the partially-silvered end.

From the 18.70 eV level the Ne atom passes down spontaneously to a lower metastable state emitting incoherent light, and finally to the ground state through collision with the tube walls. The final transition is thus radiationless.

Actually, there are other energy levels between the 20.66 eV and 18.70 eV levels in Ne atom, and transitions between them result in the emission of radiation having wavelengths 3.39 μm , 1.15 μm , besides the visible radiation of wavelength 0.6328 μm (6328 Å).

Typical power outputs of He-Ne lasers lie between 1 and 50 mW of continuous wave for inputs of about 5-10 W.

He-Ne laser is a 'tunable' laser. The He-Ne laser can be tuned (adjusted) to give radiation in any desired ~~wo~~ range. This can be done by choosing mirrors having reflectivity over only the required wavelength range.