

# Logarithms and Anti-Logarithms

Suppose you are [multiplying a number](#) three times. What are you doing? You will say you will get a cube of a number if it is multiplied three times, Right? Do you know the inverse process of a cube? Finding the cube root of a number is the inverse of getting a cube. Does it ever come to your mind that how will you always get the  $1/n^{\text{th}}$  root? Isn't it very time-consuming and tiring to do so? In this section, we will learn a new concept of Logarithms and Anti-Logarithms (antilog). Antilog has a wide application in the [field of mathematics](#).

## Logarithms and Anti-Logarithms

It is not always possible to handle the numbers which are either too large or too small. To make long, tedious and confusing calculations simple, we change the form of the number using logarithms. The changed number can be put into original form by using antilog. Logarithms and Anti-Logarithms are the inverses of each other. Let us study logs and antilog in detail.

### Logarithms

A logarithm of a number is the power to which a given base must be raised to obtain that number. The power is sometimes called the [exponent](#). In other words, if  $b^y = x$  then  $y$  is the logarithm of  $x$  to

base  $b$ . For example, if  $2^4 = 16$ , then 4 is the logarithm of 16 with the base as 2. We can write it as  $4 = \log_2 16$ .

### What are Exponents?

$$b^x = y$$

$$\text{or, } x \log b = \log y$$

$$\text{or, } x = \log_b y$$

Here,  $y > 0$ ,  $b > 0$ , and  $b \neq 1$ .

### ***Logarithmic Laws and Properties***

#### **Theorem 1**

The logarithm of the product of two numbers say  $x$ , and  $y$  is equal to the sum of the logarithm of the two numbers. The base should be the same for both the numbers.

$$\log_b (x y) = \log_b x + \log_b y$$

**Proof:** Let  $\log_b x = p$  such that  $b^p = x \dots$  (i), and

$\log_b y = q$  such that  $b^q = y \dots$  (ii)

Multiplying (i), and (ii), we have

$$b^p \times b^q = x \times y = b^{(p+q)} \text{ [from the law of indices]}$$

Taking log on both sides, we have,

$$\log_b x y = p + q = \log_b x + \log_b y.$$

## Theorem 2

The [division](#) of the two numbers is the antilog of the difference of logarithm of the two numbers. The base should be the same for both the numbers.

$$\log x/y = \log x - \log y$$

**Proof:** Let,  $\log_b x = p$  such that  $b^p = x \dots$  (i), and

$$\log_b y = q \text{ such that } b^q = y \dots \text{ (ii)}$$

Dividing (i) by (ii), we have

$$x/y = b^p/b^q = b^{(p-q)} \text{ [from the law of indices]}$$

Taking log on both sides, we have,

$$\log x/y = p - q = \log x - \log y$$

## Theorem 3

The logarithm of a number to any other base can be determined by the logarithm of the same number to any given base. Mathematically, the relation is

$$\begin{aligned} \log_a x &= \log_b x \times \log_a b \\ \Rightarrow \log_b x &= \log_a x / \log_a b \end{aligned}$$

**Proof:** Let,  $\log_a x = p$ ,  $\log_b x = q$ , and  $\log_a b = r$ . From the definition of logarithms, we have

$$a^p = x = b^q, \text{ and } a^r = b.$$

$$b^q = x \text{ can be written as } (a^r)^q = a^{r^q} = x.$$

Since,  $a^p = b^q = a^{r^q} = x$ . Comparing the powers, we have

$$p = r^q$$

$$\text{or, } \log_a x = \log_a b \times \log_b x$$

$$\text{or, } \log_b x = \log_a x / \log_a b.$$

#### **Theorem 4**

The logarithm of a number raised to a power is equal to the index of the power multiplied by the logarithm of the number. The base is the same in both the conditions.

$$\log_b x^n = n \log_b x.$$

**Proof:** Let  $\log_b x = p$  so that  $b^p = x$ . Raising both sides to power  $n$ , we have

$$(b^p)^n = x^n \Rightarrow b^{pn} = x^n$$

Taking log on both the sides, we have  $\log_b x^n = pn$

$$\text{or, } \log_b x^n = n \log_b x.$$

- $\log_b (x + y) = \log_b x + \log_b (1 + y/x)$
- $\log_b (x - y) = \log_b x + \log_b (1 - y/x)$

### ***Logarithmic Table***

It is not always necessary to find the logarithm of a number by mere calculation. We can also use logarithm table to find the logarithm of a number. The logarithm of a number comprises of two parts. The whole part is the characteristics and the decimal part is the mantissa.



### ***Positive Characteristic***

The whole part or the integral part of a number is the characteristic.

The characteristic of the logarithm of any number greater than 1 is positive and is one less than the number of the digits to the left of the decimal point in the given number. If the number is less than one, then the characteristic is negative and is one more than the number of zeros to the right of the decimal point.

### ***For Example***

<b>Number</b>	<b>Characteristic</b>
4	0 [one less than the number of digits to the left of the decimal point].
21	1
111	2
0.1	- 1 [one more than the number of zeros on the right immediately after the decimal point].
0.025	- 2
0.000010	- 5



### **Negative Characteristic**

The logarithm of a number having 'n' zeros immediately after the decimal is  $-(n + 1) +$  the decimal.

### **Mantissa**

The decimal part of the number logarithm of a number is the mantissa. A mantissa is always a positive quantity. The negative mantissa should always be converted into a positive one. For example,  $-5.2592 = -6 + (1 - 0.2592) = \bar{6} + 0.7428$

### **Anti-Logarithms (Antilog)**

The anti-logarithm of a number is the inverse process of finding the logarithms of the same number. If  $x$  is the logarithm of a number  $y$  with a given base  $b$ , then  $y$  is the anti-logarithm of (antilog) of  $x$  to the base  $b$ .

**If  $\log_b y = x$  then,  $y = \text{antilog } x$**

Natural Logarithms and Anti-Logarithms have their base as 2.7183. The Logarithms and Anti-Logarithms with base 10 can be converted into natural Logarithms and Anti-Logarithms by multiplying it by 2.303.

### **Anti-Logarithmic Table**

To find the anti-logarithm of a number we use an anti-logarithmic table. Below are the steps to find the antilog.

- The first step is to separate the characteristic and the mantissa part of the number.



- Use the antilog table to find a corresponding value for the mantissa. The first two digits of the mantissa work as the row number and the third digit is equal to the column number. Note this value.
- The antilog table also includes columns which provide the mean difference. For the same row of the mantissa, the column number in the mean difference is equal to the fourth digit. Note this value.
- Add the values so obtained.
- In the characteristic add one. This value shows the place to put the decimal point. The decimal point is inserted after that many digits from the left.



Learn more about [Logarithmic Differentiation here in detail](#).

## Solved Examples on Logarithms and Anti-Logarithms

**Problem:** Find the value of  $\log 2.8726$ .

**Solution:** Here the number of digit to the left of the decimal is 1 so the value of the characteristic will be one less than one i.e., 0. From the log table, the value of 2.8726 is 0.45827. Adding the values of mantissa and the characteristic we find the value of the logarithm. So,  $\log 2.8725 = 0 + 0.45827 = 0.45827$ .

**Problem:** Calculate the antilog of 3.6552.

**Solution:** Here we need to find the number whose logarithm is 3.655. From the antilog table, the value corresponding to the row 65 and column 5 is 4508. The mean difference column for the value 2 is 2. Adding these two values, we have  $4518 + 2 = 4520$ . The decimal point is placed in  $3 + 1 = 4$  digits from the left. So,  $\text{antilog } 3.6552 = 4520.0$