Ohms. Law

For a conductor, such as metal, a linear relationship exists betn potential difference of two ends of the conductor and current flowing betn them, provided the temperature and other physical conditions remain unaltered.

This is familier ohm's law

For a straight wire of uniform cross-section whose ends are kept at a potential difference 'v', ohm's law takes the form.

$V = R.I$ $I \rightarrow$ Current flowing through the wire
R is a constant. Called resistance.

$\therefore$ ohm $= \frac{\text{volt}}{\text{ampere}} = \frac{V \times C}{C \times A} =$

Capacitance $= \frac{Q}{V}$

If 'l' is length and 'α' is cross-sectional area of a wire, the resistance 'R' is given by

$R = \rho.\frac{l}{A}$ $\rho \rightarrow$ (Resistivity, depends on material of the wire).

Unit of $\rho =$ ohm $\times \frac{m^2}{m} =$ ohm $\times$ m

Again, $V = RI$

or, $I = \frac{1}{R}.V = GV$ | $G = \frac{1}{R}$ is called conductance
Unit of conductance $=$ mho $= \Omega^{-1}$ or Siemens.

For a homogeneous, isotropic wire the electric field 'E' inside it is longitudinal and will have same value at all points along the wire.

$\therefore V = E.L$

As, $V = RI = EL$

or, $\rho\frac{L}{A}.I = E.L$

$\therefore E = \rho.\frac{I}{A} = \rho J$ $\therefore$ 'J' is current density.

$\therefore \boxed{J = \frac{E}{\rho} = \sigma E}$ $\therefore$ 'σ' is conductivity of the material.

$\rightarrow$ Differential form of ohm's law Unit of 'σ' $\rightarrow$ mho/m or Siemens/m

vectorially, $\boxed{\vec{J} = \sigma\vec{E}}$

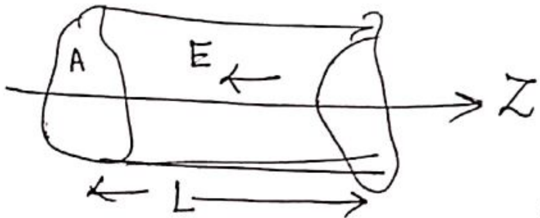
Metallic conductors have resistivity $\sim 10^{-8}$ ohm-m

" thus negligible electric field is required to drive them.

"Resistors" by constrast are made of poorly conducting material

Q1) A cylindrical resistor of cross-sectional area 'A' and length 'l' is made from material with conductivity 'σ'. If potential is constant over each end, and potential difference betⁿ the ends is 'V', what current flows?

Solⁿ

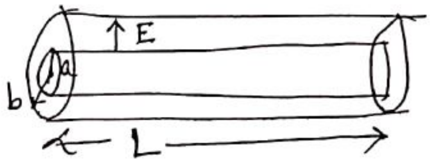


Electric field is uniform in the wire thus current density is uniform.

$$\therefore\ I = \underbrace{J \cdot A = \sigma E}_{\text{ohm's law}} \cdot A = \sigma \cdot A \cdot \frac{V}{L}$$

Q2) Two long cylinders (radii 'a' and 'b') are separated by material of conductivity 'σ'. If they are maintained at a potential difference 'V', what current flows from one to the other, in a length L?

Solⁿ



Electric field betⁿ the cylinders is

$$E = \frac{\lambda}{2\pi\epsilon_0 s}\,\hat{s}$$, 'λ' is charge per unit length of inner cylinder.

$$\therefore\ I = \int J \cdot da = \sigma \int E \cdot da = \sigma \int_0^L \frac{\lambda}{2\pi\epsilon_0 s}\, \underbrace{(2\pi s)\,dl}_{\text{area of elementary cylinder.}}$$

$$\boxed{I = \frac{\sigma}{\epsilon_0}\lambda L}$$ ✓

$$\therefore\ V = -\int_b^a E \cdot dl$$

$$= \frac{-\lambda}{2\pi\epsilon_0} \ln(a/b)$$

$$\boxed{V = \frac{\lambda}{2\pi\epsilon_0} \ln(b/a)}$$

$$\therefore\ \frac{V}{I} = \frac{\cancel{\lambda}}{2\pi\cancel{\epsilon_0}} \ln(b/a) \times \frac{\cancel{\epsilon_0}}{\sigma \cancel{\lambda} L}$$

$$\boxed{\frac{V}{I} = \frac{1}{2\pi\sigma L} \ln(b/a)}$$ ✓

For steady current ~~of~~ and uniform. conductivity.

$\boxed{3}$

$J = \sigma E$ (ohm's law. holds)

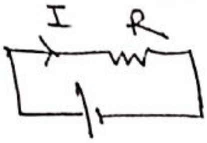
$\therefore\ E = J/\sigma$

$\therefore\ \vec{\nabla}.\vec{E} = \frac{1}{\sigma}\left(\vec{\nabla}.\vec{J}\right) = 0$

$\therefore$ charge density $(\vec{J})$ is Zero.

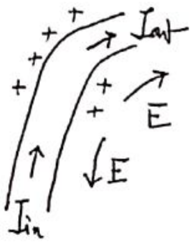
and any unbalanced charge resides on the surface.

Electromotive Force (E.m.f)?



Why is the current is same all the way around the loop?

Ans:- If current is not same all the way around, then charge is piling up somewhere — thus electric field of this accumulating charge is in such a direction as to even out the flow.



Say, current into the bend is greater than current out. Thus, charges piles up at the "Knee", this produces field aiming away from Kink. This field opposes current flowing in. (thus, slows it down) and promotes current flowing out. (speeds it up) untill currents are equal.

∴ The upshot is there are really two forces involved in driving current around a circuit ① The source 'f_s' (confined to one portion of loop) a battery say
② Electrostatic force serves to smooth out the flow and communicate the influence of source to distance parts of ckt.

$$\therefore \; f = f_s + E$$

Caused by Battery / Piezoelectric crystal / thermocouple / photoelectric cell.
In mechanism its net effect is determined by line integral of 'f' around ckt.

$$\therefore \; \boxed{\mathcal{E} \equiv \oint \vec{f}\cdot\vec{dl} = \oint f_s \cdot \vec{dl}}$$

As, $\oint \vec{E}\cdot\vec{dl} = 0$
for electrostatic field
$\therefore \oint \vec{f}\cdot\vec{dl} = \oint \vec{f_s}\cdot\vec{dl}$

'$\mathcal{E}$' is electromotive force or emf
i.e integral of force per unit charge.

Within an ideal source of emf net force on the charges is 'Zero'.
ie for $\sigma = \infty$. | Ideal source doesn't have internal resistance in real battery $|f_s| > |F_{ab}|$.

Inside ideal battery $\therefore$ $E = -f_s$ (Potential difference betn terminals (a and b))

$\therefore V = -\int_a^b \vec{E}\cdot\vec{dl}$

$\int_a^b f \cdot dl = 0$ inside battery As electrons don't move through battery.

$= \int_a^b \vec{f_s}\cdot\vec{dl}$

$= \oint \vec{f_s}\cdot\vec{dl} = \mathcal{E}$

We can extend the integral to entire loop because, $f_s = 0$, outside source.

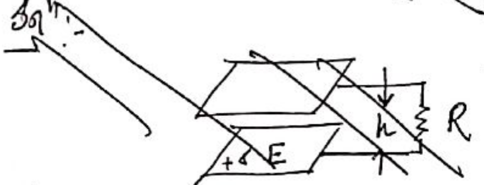
The function of a battery is to establish and maintain voltage difference = Electromotive force.

As, $\mathcal{E} = \oint \vec{f_s}\cdot\vec{dl}$ → (emf is betn two terminals over external ckt)

'$\mathcal{E}$' (electromotive force) is interpreted as work done per unit charge by the source.

Q) A battery of emf '$\mathcal{E}$' and internal resistance 'r' is hooked up to a variable "Load" resistance 'R'. If you want to deliver maxm possible power to load, what should ~~be the~~ its resistance 'R' you choose?

Soln:-



~~A rectangular loop of wire is situated so that one end is betn plates~~

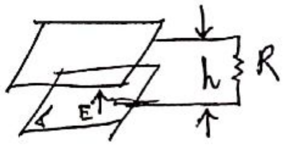
Soln:- $I = \dfrac{E}{(r+R)}$; $P = I^2 R = \dfrac{\mathcal{E}^2}{(r+R)^2}\cdot R$

$$\frac{dP}{dR} = \mathcal{E}^2\left[\frac{1}{(r+R)^2} - \frac{2R}{(r+R)^3}\right] = 0$$

$$\therefore r + R = 2R \Rightarrow \boxed{r = R} \text{ soln}$$

Q) A rectangular loop of wire is situated so that one end (height h) is betn the plates of a parallel-plate capacitor oriented parallel to the field $\vec{E}$. The other end is way outside, where the field is essentially Zero. What is the emf in this loop? If the total resistance is 'R', what current flows? Explain

Soln

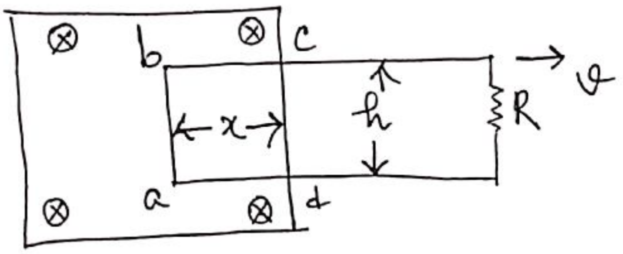


$\mathcal{E} = \oint \vec{E}\cdot\vec{dl} = 0$ for all electrostatic fields.
It looks as, $\mathcal{E} = \oint \vec{E}\cdot\vec{dl} = \frac{\sigma}{\epsilon_0} h$ as would indeed be the case if field were really just 'σ/ϵ_0' inside and Zero outside.

Actually, there is a "fringing field" at edges which is just right to Kill off contribution from the left end of loop. Thus current is Zero.

Motional emf

Generators exploit "motional emf's". which arise when you move a wire through a magnetic field.



Uniform magnetic field pointing into the page in SQUARE region.

'R' → resistance of loop.

If entire loop pulled to the right, with speed $\vec{\vartheta}$.

i) Charges in segment 'ab' experiences magnetic force, whose vertical component '$q\vartheta B$' drives current around the loop in clockwise direction.

The e.m.f. is

$$\mathcal{E} = \oint \vec{f}_{mag} \cdot d\vec{l} = \vartheta B h$$

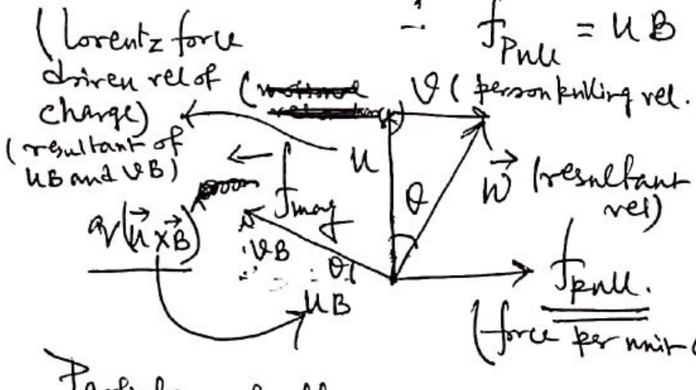
where $\vec{f}_{mag}$ = (force per unit charge) $= q(\vec{\vartheta} \times \vec{B})$ with $q = 1$

When current flows charges in segment 'ab' have a vertical velocity (say u) in addition to ϑ (horizontal)

∴ Magnetic force has a component quB to left $\Leftarrow q(\vec{u} \times \vec{B})$

To counteract this person pulling wire must exert force per unit charge

∴ $f_{pull} = uB$



∴ The person pulling the loop supplies energy to heat the loop. not the magnetic force As it is no work force

Particle actually moving in dirn of resultant vel 'W'

$\cos\theta = \frac{u}{W}$, $\frac{h}{\cos\theta} = h \times \frac{W}{u}$ } → Distance particle goes

∴ Work done per unit charge

$$= \int f_{pull} \cdot dl = (uB) \cdot \left(\frac{h}{\cos\theta}\right) \cdot \cos(90-\theta)$$

↓ angle betn f_{pull} and wire

$$= \cancel{u} B \times h \times \frac{\cancel{W}}{\cancel{u}} \times \frac{\vartheta}{\cancel{W}}$$

$$= Bh\vartheta = \mathcal{E} = \text{emf}$$

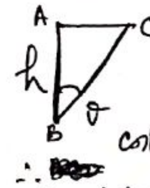
f_{pull} → Contributes nothing to emf as $\perp^r$ to wire

f_{mag} → Contributes nothing to work as $\perp^r$ to charge motion.

Component of 'u' along W

As, h and u directed same

∴ $W\cos\theta = u$



$\cos\theta = \frac{AB}{BC}$

∴ $BC = \frac{AB}{\cos\theta}$

Flux Rule

Emf generated in the loop is minus the rate of change of flux through the loop.

$$\boxed{\mathcal{E} = -\frac{d\Phi}{dt}}$$

Proof :-

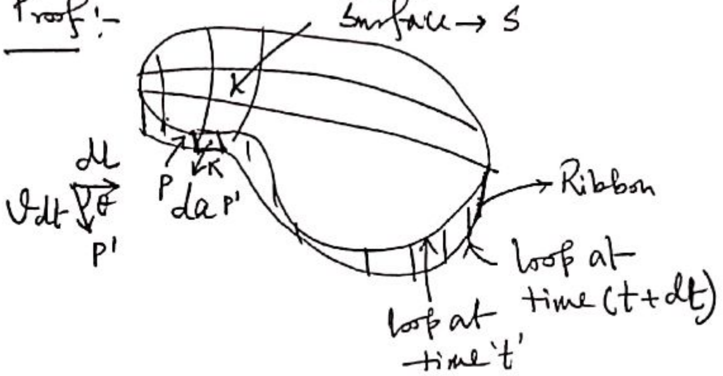


Figure shows a loop of wire at time 't' also a short time 'dt' later. Suppose we compute flux at time 't', using surface 'S' and flux at time 't+dt', using surface ~~containing~~ consisting of S- plus ribbon that connects new position of loop to old.

change in flux

$$d\Phi = \Phi(t+dt) - \Phi(t)$$
$$= \Phi_{ribbon} = \int_{ribbon} B \cdot da.$$

For point 'P' in time 'dt' it moves to P'. Let $\underline{v}$ be vel of wire, 'u' is velocity of charge down the wire

$\therefore$ $w = v + u$ ← resultant velocity of a charge at 'P'.

$\therefore$ Infinitesimal element of area on ribbon

$$da = (v \times dl)\, dt$$

$$\therefore \frac{d\Phi}{dt} = \oint \vec{B} \cdot (\vec{v} \times \vec{dl})$$

Since, $w = v + w$ and u is parallel to $\underline{dl}$

$$\therefore \frac{d\Phi}{dt} = \oint \vec{B} \cdot (\vec{w} \times \vec{dl})$$

Now, $\vec{B} \cdot (\vec{w} \times \vec{dl}) = -(\vec{w} \times \vec{B}) \cdot \vec{dl}$

$$\therefore \frac{d\Phi}{dt} = -\oint \underbrace{(\vec{w} \times \vec{B})}_{\text{magnetic force pr unit charge}} \cdot \vec{dl}$$

$$= -\oint \vec{f}_{mag} \cdot \vec{dl} = \mathcal{E}$$

$$\therefore \boxed{\mathcal{E} = -\frac{d\Phi}{dt}}$$