Sem-2 (General)

Electromagnetic induction :– Faradays laws of electromagnetic induction. Lenz's law, Self and mutual inductance. L of single coil. M of two coils.

Q1) ফ্যারাডের তড়িৎচুম্বকীয় আবেশের সূত্রগুলি বিবৃত কর
State the laws of electromagnetic induction.

Q2) Determine self inductance of a air core solenoid of length 'l', and '2r' diameter, having 'N' number of turns.
একটি বায়ুঅন্তর্যুক্ত 'l'm লম্বা ও '2r'm ব্যাসের, 'N' পাকযুক্ত সলিনয়েডের স্বাবেশাঙ্ক নির্ণয় কর,

Q3) Define self inductance of a coil. What is its unit?
কুণ্ডলীর ক্ষেত্রে স্বাবেশাঙ্কের সংজ্ঞা দাও, এর একক কী?

Q4) Calculate the coefficient of mutual inductance of two coaxial solenoids.
দুটি সমাক্ষ সলিনয়েডের পারস্পরিক আবেশ গুণাঙ্ক নির্ণয় কর,

Q5) How is non-inductive coil constructed?
আবেশহীন কুণ্ডলী কিভাবে গঠন করা যায়?

Q6) Calculate the coefficient of self inductance for two long parallel wires.
দুইটি দীর্ঘ সমান্তরাল তারের ক্ষেত্রে স্বাবেশাঙ্কের মান গণনা কর,

Q7) For two magnetically coupled coils having self inductances 'L_1' and 'L_2' ~~and mutual inductance M~~ carrying currents find the relation between self inductance and mutual inductance and find the maximum value of the mutual inductance.

Q8) Calculate the energy of the system of two coils having self inductance L_1 and L_2 and mutual inductance 'M' carrying steady current 'I_1' and 'I_2' respectively.

Q9) Starting from energy stored in the magnetic field of a coil carrying current, derive the energy per unit volume in a magnetic field.

4

ELECTROMAGNETIC INDUCTION

We know that when current starts passing through a wire a magnetic field is produced in the space surrounding the wire. But does current starts flowing through a wire when it is placed in a magnetic field? We know that when a current carrying wire is placed in a magnetic field, it experiences a force and moves. But can current flow in a wire if it is moved in a magnetic field ?

Only experiments can tell the truth.

Faraday conducted a series of experiments to study such phenomena.

4.1 Faradays experiment

In Fig.4.1, *C* is a coil of wire connected to a sensitive galvanometer *G*, *NS* is a bar magnet held near to the coil. Obviously magnetic flux from the

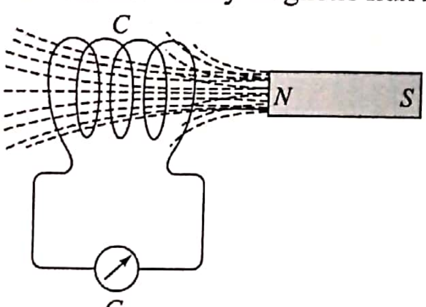


Fig. 4.1

bar magnet passes through or linked with the coil. If the distance between the two decreases, the magnetic flux linked with the coil increases and vice versa. If there is current in the coil, galvanometer shows a deflection. Current flows in the circuit only if an emf is induced in the circuit.

Faraday made the following observations and arrived at important conclusions.

1. When the coil and the magnet are *at rest*, there is *no deflection* in the galvanometer.

So if magnetic flux linked with a coil *does not change with time no emf* is induced in the coil.

2. If any *one* of them, magnet or coil, is kept *fixed* in its position and the *other* is *moved* towards and away from it, then deflections occur in the galvanometer in *opposite* directions. If the two (coil and magnet) are moved with different velocities, then also a deflection is produced in the galvanometer.

So, whenever there is *relative velocity* between a magnet and a coil i.e., magnetic flux linked with a coil *changes with time*, an emf is induced in the coil.

3. If the *speed* of approach or separation of the magnet and the coil is *increased*, the deflection *increases proportionally* and if the speed is decreased the deflection decreases.

So, the *induced emf is proportional to the time rate of change of magnetic flux linked with the circuit.*

4. Direction of current was observed carefully for different motions of the magnet and the coil.

He arrived at the conclusion that the direction of the induced emf is such that it opposes any change of magnetic flux in the circuit. Induced emf tends to *decrease* the flux linked to a coil when magnetic flux linked to it is *increasing* and vice versa.

He performed many other similar experiments with electromagnets, instead of magnet. Lenz and Henry also independently made similar experiments and observations.

The phenomenon thus discovered is known as electromagnetic induction. We shall now study the basic laws of this phenomenon.

4.1.1 *Faraday's law and Lenz's law*

Faraday's law **:** Whenever there is a relative velocity between a magnet and a coil or the magnetic flux linked with a coil changes with time, an emf is induced in the coil. The induced emf ($\in$) is proportional to the time rate of change of magnetic flux (Φ_m) linked with the coil.

$$\in \propto \frac{d\Phi_m}{dt}$$

Lenz's law : The direction of the induced emf is such as to oppose the cause of induction.

Combining the two laws we get

$$\in = -\frac{d\Phi_m}{dt} \quad ....(4.1)$$

Negative sign comes from Lenz's law that $\in$ always opposes the change of flux. In SI unit the proportionality constant comes out to be 1.

Unit of $\frac{d\Phi_m}{dt}$ is $\frac{\text{Wb}}{\text{s}} = \frac{\text{N}\cdot\text{m}}{\text{A}\cdot\text{s}} = \frac{\text{J}}{\text{C}} = \text{V}$.

If there are N turns in a coil and magnetic flux linked with each turn is Φ_m, the total flux linked or *flux linkage* is $N\Phi_m$ the above eqn. becomes

$$\in = -N\frac{d\Phi_m}{dt} \quad ....(4.2)$$

The eqn. 4.1 or 4.2 is the basic form of Faraday-Lenz's law.

By changing magnetic flux linked with a coil periodically with time, therefore, we can generate periodically changing emf. This fact is utilised in the construction of transformer.

4.1.2 *Lenz's law and conservation of energy*

Lenz's law asserts that induced emf opposes the change of flux. To produce change of flux, therefore, some external agent must do the necessary work against the induced emf. Hence we get the induced emf by doing work. Thus Lenz's law supports the principle of conservation of energy.

4.1.3 *Faraday's law in integral and differential forms*

In Fig. 4.2 C is a single turn of wire or a coil. Magnetic flux (Φ_m) pases through it. If the magnetic flux changes with time an emf ($\in$) is induced in C.

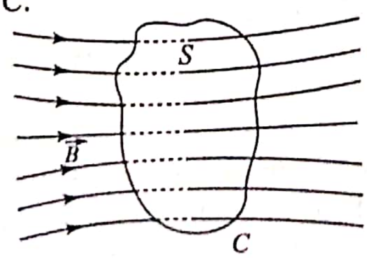


Fig. 4.2

We know emf is the line integral of an electric field in a closed path. Therefore when an emf $\in$ is induced in a circuit, associated electric field $\vec{E}$ is also induced in it.

The emf $\in$ of the above eqn. 4.1 can be written as the line integral of the induced electric field $\vec{E}$ around the coil.

$$\in = \oint_C \vec{E}\cdot d\vec{l}$$

Here C is the boundary curve of the coil.

The magnetic flux Φ_m can be written as the surface integral of the magnetic field $\vec{B}$:

$$\Phi_m = \int_S \vec{B}\cdot d\vec{a}$$

Here S is the open surface enclosed by the curve C.

Substituting these two in eqn.4.1 above, we get

$$\oint_C \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\left(\int_S \vec{B}\cdot d\vec{a}\right)$$

If the circuit is *fixed*, the time derivative can be moved inside the surface integral and then it becomes *partial* derivative, because magnetic field *may vary in space*. The equation becomes

$$\oint_C \vec{E}\cdot d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t}\cdot \hat{n}da \quad ....(4.3)$$

This is the *integral form* of Faraday's law.

By Stokes' law the line integral of electric can be written as the surface integral of its curl and so the above eqn. becomes

$$\oint_C \vec{E}\cdot d\vec{l} = \int_S (\vec{\nabla}\times\vec{E})\cdot\hat{n}da = -\int_S \frac{\partial \vec{B}}{\partial t}\cdot\hat{n}da$$

Since the above equation holds for any arbitrary surface S, we get

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad ....(4.4)$$

This is the *differential form* of Faraday's law.

From eqns. 4.3 and 4.4, we see that the line integral of the induced electric field over a closed path and its curl are *non-zero*. Hence this is not of the same nature as that of the electrostatic field. *It is non-electrostatic field.* Also we notice that *this field is produced by a time-varying magnetic field.*

This is indeed a fundamental discovery of Faraday that *when magnetic field varies in time an electric field is produced.* We came across a *new phenomenon* or a new relation between electric and magnetic fields, which is not revealed in magnetostatics.

4.1.4 *Motional emf*

Now we shall study another discovery of Faraday that *if a conductor is moved through a magnetic field, cutting the field lines, an electromotive force is induced across its ends.* This is called *motional emf.*

This phenomenon, however, *can be predicted* from other known laws of magnetostatics. Hence it is not a new phenomenon. Now let us see how the emf arises and what its value is.

We take a simple situation. There is a constant magnetic field $\vec{B}$ along the *Z*-axis, Fig.4.3. A piece

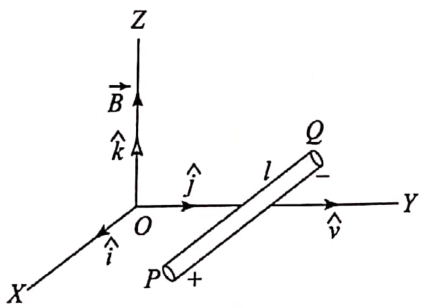


Fig. 4.3

of conducting rod *PQ* is lying parallel to the *X*-axis. It starts moving along *Y*-axis with a constant velocity *v*.

$$\therefore \vec{B} = B\hat{k} \text{ and } \vec{v} = v\hat{j}$$

When the conductor moves, the free electrons in it also get the motion. So, each electron moving in the magnetic field will experience the Lorentz force

$$\vec{F}_m = (-e)\vec{v} \times \vec{B} = (-e)vB(\hat{j} \times \hat{k})$$
$$= -\,evB(\hat{i}\,) = -\,evB\,\hat{i}$$

By the action of this force, free electrons in the conductor start moving towards the end *Q* and accumulate there and deficit of electrons grows at the end *P*. *P* gets positive potential and *Q* gets negative potential. As a consequence of this, an electric field $\vec{E}$ develops which acts from *P* to *Q*. The Lorentz electric force on a free electron due to this electric field $\vec{E}$ is

$$\vec{F}' = (-e)\vec{E} = -e(-E\hat{i}) = eE\hat{i}$$

When the two opposite forces $\vec{F}_m$ and $\vec{F}'$ become equal, the motion of electrons towards *Q* stops. This happens when

$$evB = eE \;\therefore\; E = vB.$$

If the electric potentials of *P* and *Q* are V_P and V_Q and length of the conductor *PQ* is *l*, we can write

$$V_P - V_Q = E \cdot l = vBl.$$

Thus we find a very interesting phenomenon: When a conductor of length *l* moves with velocity *v at right angle* to a magnetic field *B*, a potential difference develops between the two ends of the conductor. The value of the potential difference is equal to $\epsilon_m = vBl$. This is called *motional emf.*

If we join the two ends of the moving rod *PQ* to a tiny bulb it would glow, Fig. 4.4. The moving

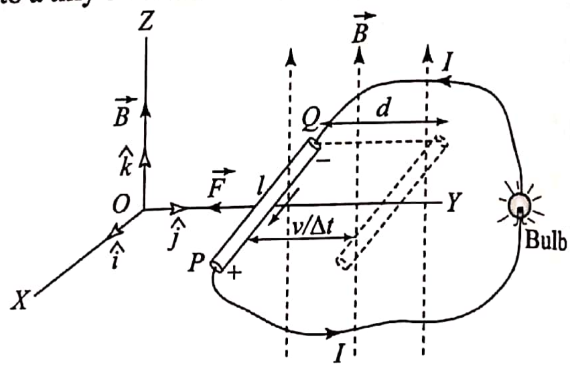


Fig. 4.4

rod can supply current just like an electric cell. Now the question is where the energy coming from when it sends current.

To get the answer, we suppose that current flowing in the circuit is *I*. As it flows through the external circuit, it also flows through the conductor from *Q* to *P*. The conductor is now carrying current *I* as it moves through the magnetic field *B*. Therefore the conductor experiences a force

$$\vec{F} = I\vec{l} \times \vec{B} = IlB(\hat{i} \times \hat{k}) = -IlB\hat{j}$$

The direction of the force is opposite to the direction of the motion.

Hence in order to move the rod with a constant velocity someone must pull the rod with the same force in the positive direction of *Y*-axis. Hence he is supplying the energy necessary to send the current. Thus *mechanical energy lost by the external agent is converted to electric energy.* This is a very satisfactory result; it upholds the principle of conservation of energy. Work is to be done to get energy.

Now we proceed to get a more useful expression for the motional emf.

Work done by the motional emf ϵ_m to send current *I* for time Δt is

$$\Delta W = \epsilon_m I \Delta t\,.$$

During that time the rod has moved through a distance $\Delta d = v\Delta t$. The work done by the external agent or loss of energy of the external agent during that time is

$$\Delta W' = -F\Delta d = -IlB\Delta d.$$

By law of conservation of energy we have $\Delta W' = \Delta W$.

$\therefore \in_m I\Delta t = -IlB\Delta d$

or, $\in_m \cdot \Delta t = -l \cdot B \cdot \Delta d$ ....(i)

Now area swept over by the rod in time Δt is $\Delta a = l\Delta d$. Magnetic field is flux density.

$\therefore$ Magnetic field lines or flux the rod cuts through in time Δt is $\Delta\Phi_m = B\Delta a = Bl\Delta d$.

$\therefore$ From eqn.(i) we can write, $\in_m = -\frac{\Delta\Phi_m}{\Delta t}$

We put the condition $\Delta t \to 0$ to see what happens at each instant. We get :

Motional emf, $\in_m = -\frac{d\Phi_m}{dt}$ ....(4.5)

We find that motional emf is equal to the rate at which a *conductor cuts through magnetic flux*. In fact this is the basic principle of electric generator.

In a generator conducting wire of different shapes are moved or rotated in a constant magnetic field by different sources of energy and electric energy is generated by producing motional emf. Since here we get a potential difference from other source of energy, we call it emf.

Now we observe that eqns.4.4 and 4.5 are identical in form and both are called Faraday's law, but these two represent two different facts.

Let us recapitulate the difference: The first equation essentially tells that a time varying magnetic field produces an electric field; the second one tells that an emf develops across the ends of a conductor, moving in a magnetic field.

4.1.5 *EMF induced at the ends of a conductor rotating with a uniform angular velocity at right angle to a uniform magnetic field*

We consider a conductor of length L rotating at right angle to a uniform magnetic field B, Fig.4.5a. As it is moving in a magnetic field cutting through magnetic flux, motional emf will be induced across the ends of the conductor. We notice that linear velocity is different for different portions of the conductor, so we have to calculate the emf in terms of angular velocity. We consider an element of the conductor of length dx at a distance x from the centre O. EMF developed across that element of the conductor is $de = Bvdx = B\omega xdx$.

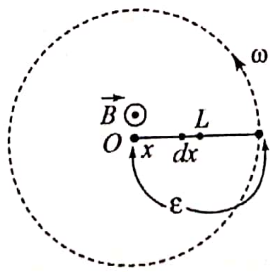


Fig. 4.5(a)

$\therefore$ Total emf is $\in = \int_0^L B\omega xdx = \frac{1}{2}B\omega L^2$.

If n be frequency of rotation of the rod, then $\omega = 2\pi n$. EMF, $\in = n\pi L^2 B$.

If instead of a conducting rod, we are given a conducting disc of radius L rotating in the magnetic field B with the same frequency n, Fig.4.5b then we can imagine the rod as a radius of the disc. The rate of cutting magnetic flux is the same and therefore the emf induced between the rim of the disc and its centre is the same as above. Notice that all points on the rim are at the same potential. Such a device can produce emf by doing work in rotating the disc. Notice some external agent must supply the energy to rotate the rod or the disc, because by Lenz's law induced emf opposes the rotation. This device is called homopolar dynamo, though it has no commercial value as a generator. But all real generators work on this principle.

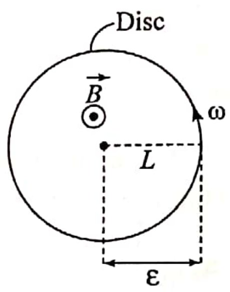


Fig. 4.5(b)

4.1.6 *Flow of charge when magnetic flux changes in coil*

There is coil of N turns connected to a closed circuit; total resistance of the circuit is R. A magnetic flux is linked with the circuit. Now suppose the flux linked to the circuit undergoes a quick change from Φ_{m_1} to Φ_{m_2} in time t. We like to know how much charge flows through the circuit during this time as a result of this.

EMF induced as result of magnetic flux change is by equation 4.2

$$\in = -N\frac{d\Phi_m}{dt}$$

Here Φ_m is the flux through a single turn. Current induced in the circuit is

$$I = \frac{\in}{R} = \frac{N}{R}\frac{d\Phi_m}{dt}$$

Charge flown in time t is

$$Q=\int_0^t Idt=-\frac{N}{R}\int_{\Phi_i}^{\Phi_f} d\Phi_m$$

$$=-\frac{N(\Phi_f-\Phi_i)}{R}=\frac{\Phi_{m_1}-\Phi_{m_2}}{R} \quad ....(4.6)$$

We notice that the amount of charge induced by the change of flux *does not depend upon the rate of change of flux*, but depends on the resistance of the circuit and change of flux.

4.1.7 *Measurement of charge by ballistic galvanometer*

When a sudden change of magnetic flux occurs in a circuit, some charge flows through the circuit within a very short time. This induced charge can be measured by ballistic galvanometer. Ballistic galvanometer is nothing but ordinary suspended galvanometer with the following modifications.

The requirement of a ballistic galvanometer :

1. *Time period* of the coil of the galvanometer should be *sufficiently large* compared to the time taken by the induced charge to pass through the circuit.

2. *Damping* of the movement of the coil should have *minimum* value.

If the first condition is satisfied, the coil practically cannot start moving before the whole charge passes through it. As the charge passes through it, the coil gets an angular impulse when it is almost at its rest position. The kinetic energy thus gained by the coil produces a deflection of the coil. It is found that the *first throw* of the coil is *proportional* to the *charge* passed through it. This first throw, however, is *reduced* by the force of damping. Hence the second requirement is important.

Time period of the moving coil is given by

$$T=2\pi\sqrt{\frac{I}{c}}$$

Moment of inertia (I) of the coil is increased by increasing the breadth of the coil and by using a very delicate suspension, so that the restoring torque per unit twist (c) is very small. Use of broader coil, on the other hand, increases the air gap between the coil and pole pieces of the galvanometer. This decreases the magnetic field. To keep the value of the field high, the soft iron core between the pole faces is made wider.

When the coil moves in the magnetic field, an induced emf develops which opposes its motion, by Lenz's law. This is called electromagnetic damping. To reduce this damping, the coil is wound on a non-conducting frame such as wood, ebonite or bamboo. Still there is inevitable damping due to air resistance. For this damping even the first throw is reduced somewhat. This effect due to damping is corrected by determining the logarithmic damping. We shall not discuss the theory of ballistic galvanometer. We only quote the result.

Charge passed through a ballistic galvanometer is given by

$$Q=\frac{cT}{2\pi nAB}\theta_1\left(1+\frac{\lambda}{2}\right)=k\theta_1\left(1+\frac{\lambda}{2}\right)$$

The constant k is the galvanometer constant.

Here c = restoring torque per unit twist of the suspension, n = number of turns in the coil, A = area of the coil, B = magnetic field, T = time period of the coil in *open circuit*, θ_1 = the first throw and λ is the logarithmic decrement.

The formula for logarithmic decrement is as follows : If $\theta_1, \theta_2, \theta_3, ...\theta_n$ be the successive amplitudes of oscillation of the coil alternately to the right and left, then

$$\lambda=\frac{1}{n-1}\log_e\frac{\theta_1}{\theta_n}.$$

To determine the logarithmic decrement, we first note the first throw (θ_1) and then go on counting the successive throws and note the 11th throw (θ_{11}), say. Both will be on the same side. Then from the above formula we can calculate λ.

4.2 Self-Inductance

We know that if a coil C of single turn or many turns carries a current I, a magnetic field B is produced within it and in the surrounding space, as shown in Fig.4.6. Hence a magnetic flux (Φ_m)

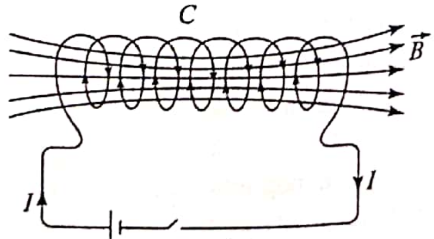


Fig. 4.6

is linked with the coil. It is called self-flux. Since magnetic field is proportional to the current, magnetic flux Φ_m is also proportional to the current.

$\therefore \Phi_m = LI$ ....(4.7)

The proportionality constant L is called the *self-inductance* or simply *inductance* of the coil. The value of L depends on the geometrical factors of the coil, like size, shape and number of turns of the coil. If instead of air or vacuum, there is a ferromagnetic material as core in the coil, self-inductance may increase more than thousand times. Here we assume the absence of such materials.

Now suppose that the current in the coil varies with time and so the magnetic flux changes with time. According to Faraday's law an emf is induced in the coil which opposes the change of current. The induced emf is given by

$$\epsilon = -\frac{d\varphi_m}{dt} = -L\frac{dI}{dt} \text{ .... (4.8)}$$

The negative sign indicates that the *emf* tends to *decreases* the *current* if it is *increasing* and emf tends to increase the current if it is decreasing. In other words, the direction of induced emf is opposite to current, if current is increasing and induced emf is in the direction of the current if the current is decreasing, as shown in Fig, 4.7. This induced emf is sometimes called back emf.

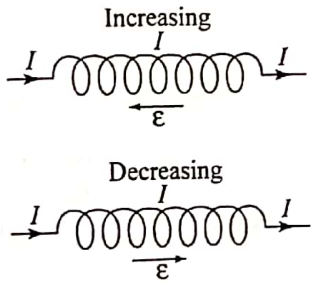


Fig. 4.7

4.2.1 *Definition of self-inductance*

From eqns.4.7 and 4.8 we can get two equivalent definitions of self-inductance :

1. Self-inductance of a coil is numerically equal to the *magnetic flux linked* with the coil when *unit* current flows through it. $\Phi_m = L$, when $I = 1$.

2. Self-inductance of a coil is numerically equal to the induced emf developed in the coil, when *time rate of change of current* in it is *unity*.

$\epsilon = L$, when $\frac{dI}{dt} = 1$.

We see from the above discussion that an electric circuit always opposes any change of current passing through it. This phenomenon is called *self-induction*. It is analogous to the property of *inertia* of matter which opposes any change of motion. Self induction occurs whenever current in a circuit changes, but is absent when a steady current flows. As a result any change in current in a circuit does not occur instantaneously.

In fact in a dc circuit self-induction occurs only when current is switched on and switched off and it is an undesirable disturbance. But in ac circuit current changes continuously and self-inductance is always present in the circuit and plays very important role.

Magnetic energy due to self-induction :

As soon as we switch on a circuit the current starts growing from zero to its final value I. This is called growth of current in a circuit. Let us see the effect of self-inductance during this growth of current in a circuit. As current increases from zero, the magnetic self flux linked with the coil increases from zero and as a result an induced emf *opposing the increase of current* develops. So, to establish the current in the circuit to its final value, the source of emf must do some *work against* this *induced emf.* Let us calculate this work.

Suppose the instant t at which current is i, induced emf is ϵ. The work done against the emf in an elementary time interval dt about t is

$$dW = \epsilon i dt = L\frac{di}{dt} i dt = Lidi$$

Current increases from 0 to final value I.

$\therefore$ Total work done against the emf is

$$W = \int_0^I Lidi = \frac{1}{2}LI^2 \text{ ....(4.9)}$$

This work done by the source of emf is *stored up in the magnetic field* established in and around the coil.

$\therefore$ Magnetic energy stored up in the coil is

$$U_m = \frac{1}{2}LI^2 = \frac{1}{2}I\Phi_m \text{ ....(4.10)}$$

From eqn.4.9, we see that $2U_m = L$, when $I = 1$. Self-inductance of a coil is *numerically equal to twice the magnetic energy* linked with the coil, when *unit current* flows through it.

When the current is switched off in a circuit the current drops to zero. This is called decay of current. The magnetic field linked with the coil collapses. As soon as the current begins to decreases self induced emf develops, which now tends to *maintain* the current by the energy stored in the magnetic field. But the current decreases exponentially to zero as the energy stored in the magnetic field is exhausted. Often a spark is observed when current is switched off; this is produced by the emf develops during decay of current in the circuit. Therefore the energy is ultimately converted to *thermal* energy.

4.2.2 *Unit and dimensions of self-inductance*

Unit of self-inductance is *henry* (H). A coil has self-inductance 1 H if (i) flux linked with the coil is one weber (Wb) when current flowing through it is one ampere or (ii) when induced emf in it is one volt, when current in it changes at the rate of one ampere per second.

$$\mathrm{H}=\frac{\mathrm{Wb}}{\mathrm{A}}=\frac{\mathrm{V\cdot s}}{\mathrm{A}}=\mathrm{VA^{-1}s}$$

Now we can get a *new unit* of μ_0 in terms of henry.

Unit of μ_0 is $\frac{\mathrm{T\cdot m}}{\mathrm{A}}=\frac{\mathrm{Wb}}{\mathrm{m\cdot A}}=\frac{\mathrm{H}}{\mathrm{m}}$

$$[L]=\frac{[\Phi_m]}{[I]}=\frac{[\mathrm{ML^2T^{-2}A^{-1}}]}{[\mathrm{A}]}=[\mathrm{ML^2T^{-2}A^{-2}}]$$

4.2.3 *Calculations of Self-inductance*

(i) Solenoid :

In the last chapter we have calculated the magnetic field inside a coil in the form of a solenoid. We have seen that if the length of the coil is much greater than the radius, the magnetic field is reasonably constant over the whole cross-section and is given by eqn. 3.18 $B=\mu_0 nI$,

where n is the number of turns per unit length and I is the current.

Suppose A= area of each coil, l = length of the coil. Then total number of turns in the coil = nl.

∴Total magnetic flux linked with the coil,

$\Phi_m = nlAB = \mu_0 n^2 lAI.$

When $I = 1$, $\Phi_m = L$.

∴ Self-inductance of the coil,

$L = \mu_0 n^2 lA = \mu_0 N^2 A/l$..........(4.11)

If N = total number of turns in the coil, $n = N/l$.

If there is ferromagnetic material as core of coil and relative permeability k_m, the self inductance would be

$L_c = k_m L$ ....(4.12)

k_m may be as large as 5000. This is a general phenomena true whenever air core is replaced by ferromagnetic materials.

In eqn. 4.12, it is assumed that the ferromagnetic core completely fills the solenoid. If it fills partly, still the flux increases and self-inductance increases. We suppose that in a solenoid the core is partly filled by one material of cross-section A_1 and relative permeability k_{m_1} and partly by another material of cross-section A_2 and relative permeability k_{m_2}. Then its self-inductance is given by

$$L = \mu_0 n^2 l(k_{m_1}A_1 + k_{m_2}A_2) \quad \text{........ (4.12a)}$$

(ii) Toroid :

We have calculated the magnetic field within an endless solenoid (also called anchor ring) in the last chapter. Let N = total number of turns, R = mean radius of the solenoid, A = area of cross-section of the solenoid and I = current flowing through the solenoid. If we suppose that the cross-section is not large, the magnetic field within it may be assumed to be constant given by eqn.3.23

$$B=\frac{\mu_0}{2\pi}\frac{NI}{R}.$$

∴ Magnetic flux linked with the coil is

$$\Phi_m = BNA = \frac{\mu_0}{2\pi}\frac{NI}{R}\cdot NA = \frac{\mu_0}{2\pi}\frac{N^2A}{R}I$$

Putting I =1, we get the self-inductance of the toroid, $L=\frac{\mu_0}{2\pi}\frac{N^2A}{R}$ ....(4.13)

(iii) Straight wire :

Self-inductance of a straight wire can be deduced in a very simple way by calculating energy.

Let R be the radius and l the length of the wire. Current flowing through it is I. The magnetic field B at a distance $r(r < R)$ from its axis, Fig.4.8, can be found by Ampere's law:

$$\int_C \vec{B}\cdot d\vec{l} = B\cdot 2\pi r = \mu_0 I',$$

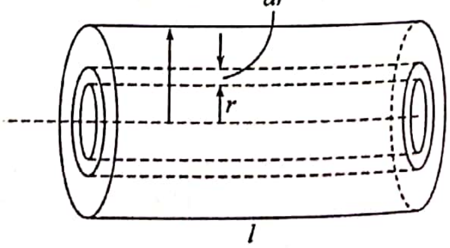


Fig. 4.8

where I' is the current enclosed by the circle of radius r. We assume that current is equally distributed over the whole cross-section of the wire, then

$$I' = \frac{\pi r^2}{\pi R^2} I \quad \therefore B = \frac{\mu_0}{2\pi r} \times I \frac{r^2}{R^2} = \frac{\mu_0}{2\pi} I \frac{r}{R^2}.$$

Magnetic field energy per unit volume associated with that magnetic field will be calculated below. From eqn.4.21

$$u_m = \frac{1}{2}\frac{B^2}{\mu_0}.$$

Substituting the value of B from above we get

$$u_m = \frac{1}{2\mu_0}\frac{\mu_0^2}{4\pi^2}\frac{I^2 r^2}{R^4} = \frac{\mu_0}{8\pi^2}\frac{I^2 r^2}{R^4}$$

We consider an elementary hollow coaxial cylinder of the conductor of radius r and thickness dr. Volume of this cylinder is $dV = 2\pi r dr \cdot l$.

Energy contained in this volume element is $u_m dV$.

$\therefore$ Total energy inside the conductor is

$$U_m = \int_0^R u_m dV = \frac{\mu_0}{8\pi^2}\frac{I^2}{R^4}\int_0^R r^2 2\pi r dr \cdot l$$

$$= \frac{\mu_0}{8\pi^2}\frac{I^2}{R^4} 2\pi l \frac{R^4}{4} = \frac{\mu_0}{8\pi}\frac{1}{2} I^2 l$$

We know that if the inductance of the wire is L and current flowing through it is I, the magnetic energy is

$$U_m = \frac{1}{2} L I^2$$

$$\therefore \frac{1}{2} L I^2 = \frac{\mu_0}{8\pi}\frac{1}{2} I^2 l$$

$\therefore$ Self-inductance of the wire is $L = \frac{\mu_0}{8\pi} l$

$\therefore$ Self-inductance per unit length of a straight wire is $\frac{\mu_0}{8\pi} = \frac{1}{2} \times 10^{-7}$ henry.........(4.14)

We note that its value is very small and it is independent of the radius of the wire.

Now let us calculate self-inductance by calculating the *flux linkage*.

The field at a distance r from the axis is

$$B = \frac{\mu_0}{2\pi} I \frac{r}{R^2}$$

Now we consider a thin cosaxial hollow cylinder of the wire having radii r and $r + dr$, Fig.4.9. The magnetic field lines are concentric circles as shown in the Fig. 4.9a. The flux passing

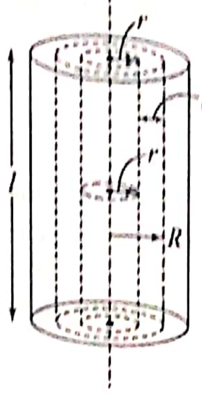


Fig. 4.9

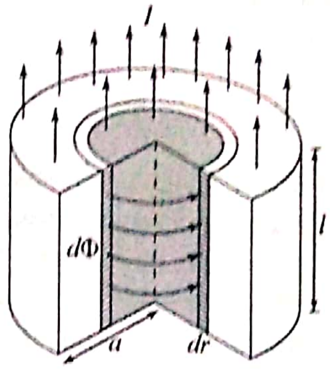


Fig. 4.9(a)

through the area $da = l \cdot dr$ due to the above field at a distance r from the axis is

$$d\Phi_m = B \cdot da = \frac{\mu_0}{2\pi} I \frac{r}{R^2} l dr$$

This flux can be considered as a tubular element that encircles a certain fraction of the total current I. This fraction is $\pi r^2 / \pi R^2$.

$\therefore$ Flux linkage corresponding to this element is

$$d\Phi'_m = \frac{\mu_0}{2\pi} I \frac{r}{R^2} l dr \frac{r^2}{R^2} = \frac{\mu_0}{2\pi}\frac{Il}{R^4} r^3 dr$$

$\therefore$ Total flux linkage

$$= \frac{\mu_0}{2\pi}\frac{Il}{R^4}\int_0^R r^3 dr = \frac{\mu_0}{2\pi}\frac{Il}{R^4}\frac{R^4}{4} = \frac{\mu_0}{8\pi} Il$$

Putting $I = 1$ and $l = 1$, we get the self-inductance of unit length of a wire is

$$L = \frac{\mu_0}{8\pi}.$$

We get the same result as above.

(iv) Two parallel wires :

We consider two parallel wires A and B separated by distance d. Current I is going away from A and coming back by B, Fig 4.10. Let radius of the each wire be r.

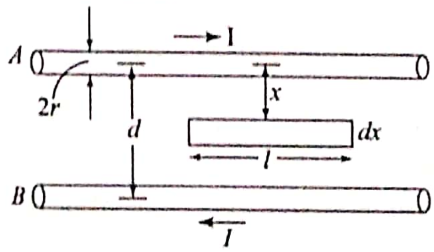


Fig. 4.10

The magnetic fields due to the two wires at any point in the space between the two wires are in the same direction. Therefore magnetic field at a distance x from upper wire is

$$B = \frac{\mu_0}{4\pi}\left[\frac{2I}{x} + \frac{2I}{d-x}\right]$$

We consider a thin area of thickness dx and length l at a distance x from the upper wire. The magnetic flux through this area is $d\Phi_m = Bldx$.

Therefore total flux through the area of length l between the two wires is

$$\Phi_m = \int_r^{d-r} Bldx = \frac{\mu_0 Il}{2\pi}\int_r^{d-r}\left[\frac{1}{x} + \frac{1}{d-x}\right]dx$$

$$= \frac{\mu_0 Il}{2\pi}[\log_e x - \log_e(d-x)]_r^{d-r}$$

$$= \frac{\mu_0 Il}{2\pi}\left[2\log_e \frac{d-r}{r}\right] = \frac{\mu_0 Il}{\pi}\log_e \frac{d-r}{r}.$$

Putting $I = 1$ and $l = 1$, we get the self-inductance per unit length of two parallel wires of radius r and separated by a distance d is

$$L = \frac{\mu_0}{\pi}\log_e \frac{d-r}{r} \quad \text{..............(4.15)}$$

We notice that self-inductance is very small if the separation between the two coils is very small, but $L = 0$ only if the two wires are in contact, $d - r = r$.

Non-inductive windings : In resistance box different standard resistances are made by coils of wire of desirable lengths. Care is taken so that these coils do not have undesirable inductances. For this purpose the insulated wire of the desirable length is first folded on itself and then wound on a bobbin, Fig. 4.10a. The current in one half flows in opposite direction to that on the other half. Physically it means that the magnetic field produced by current in one half almost cancels the field due to the current in the other half. Hence magnetic flux linked with the coil is almost zero.

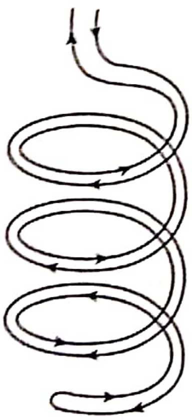


Fig. 4.10(a)

From the above calculation also, we see that one Self-inductance is reduced to a very small value. But it is not zero, unless the two touch.

(v) Two coaxial cylinders :

In Fig. 4.11 we see two coaxial cylinders, the radius of the inner cylinder is a and the inner radius

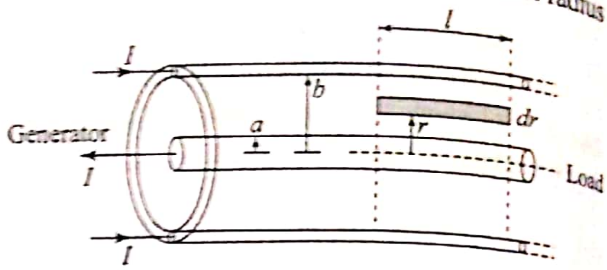


Fig. 4.11

of the outer hollow cylinder is b. Current I flows from the generator to the load through the outer cylinder and returns by the inner conductor.

The magnetic field due to the inner conductor at a distance r from the axis of the inner conductor is

$$B = \frac{\mu_0}{2\pi}\frac{I}{r}.$$

Outer conductor has no field. Therefore to find the self-inductance we consider an area of thickness dr and length l at a distance r from the axis of the inner conductor. The flux through this area is

$$d\Phi_m = \frac{\mu_0}{2\pi}\frac{I}{r}l \cdot dr$$

∴ Total flux through the area of length l in the region between the two cylinders is

$$\Phi_m = \frac{\mu_0}{2\pi}Il\int_a^b \frac{dr}{r} = \frac{\mu_0}{2\pi}Il\log_e \frac{b}{a}$$

Putting $I = 1$ and $l = 1$, we get the self-inductance per unit length of the coaxial cylinder is

$$L = \frac{\mu_0}{2\pi}\log_e \frac{b}{a} \quad \text{............(4.16)}$$

4.3 Mutual inductance

Suppose there are two coils of wire C_1 and C_2 near to each other, Fig. 4.12, and there is no

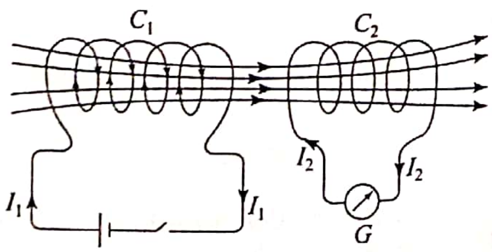


Fig. 4.12

magnetic material nearby. Current I_1 passes through C_1. Magnetic field due to C_1 at the location of the coil C_2 is proportional to current I_1. Therefore magnetic flux (Φ_{m_2}) linked with the coil C_2 is also proportional to the current I_1.

$\therefore \Phi_{m_2} = M_{12}I_1$ .... (i)

Similarly if current I_2 flows in coil C_2, magnetic flux (Φ_{m_1}) linked with the coil C_1 is given by

$\Phi_{m_1} = M_{21}I_2$ ....(ii)

It is found that $M_{12} = M_{21} = M$. This proportionality constant is called the mutual inductance of the two coils. Its value depends on the geometrical factors of the two coils and on their relative orientations and proximity. The two coils are magnetically coupled; magnetic field of one is linked with the other. So, change in current in any one induces emf in the other.

If current in any one of the two coil changes, an emf is induced in the other coil. This induced emf will produce a deflection if there is a galvanometer G joined to the other coil. For example, if current I_1 changes, emf induced in the coil C_2 is

$$\epsilon_2 = -\frac{d\varphi_{m_2}}{dt} = -M\frac{dI_1}{dt} \quad ....(4.17)$$

From the last three equations we can get two *equivalent definitions* of mutual inductance of two coils as follows:

1. Mutual inductance of two coils is numerically equal to the magnetic flux linked with one coil when current in the other is unity.

2. Mutual inductance of two coils is numerically equal to the emf induced in one coil when time rate of change of current in the other coil is unity.

4.3.1 *Mutual induction and transformer*

We see from above that change of current in one coil produces change in the current in a neighbouring coil, because magnetic flux of one coil is linked with the other. This phenomenon is called mutual induction. In most situations, this is a disturbance. We should keep different coils sufficiently far from each other to get rid of mutual induction.

Mutual induction is utilised in the construction of *transformers*. Transformer consists of two coils, wound on a common soft iron core, Fig.4.13. The core is usually made of laminated sheets placed one over the other and insulated from one

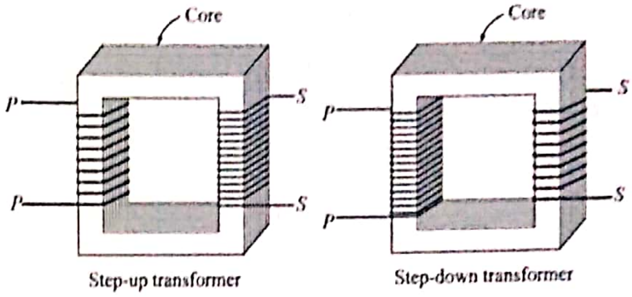


Fig. 4.13

another. The two coils are magnetically coupled; magnetic flux of one is linked with the other with minimum leakage. Alternating current is passed through one coil, which is called primary coil (P). By mutual induction alternating emf is induced in the other coil, which is called secondary coil (S). This induced emf in the secondary coil can be used as input to run any electrical devices. In an ideal transformer, it is found that

$$\frac{\text{Secondary voltage } (E_s)}{\text{Primary voltage } (E_p)} = \frac{\text{No. of turns in the secondary } (N_s)}{\text{No. of turns in the primary } (N_p)}$$

If $N_s > N_p$, $E_s > E_p$. In this condition, we get higher voltage from a lower one. Then it is called a step-up transformer.

If $N_s < N_p$, $E_s < E_p$. In this condition, we get lower voltage from a higher one. Then it is called a step-down transformer.

4.3.2 *Relation between self inductances and mutual inductance*

We consider a toroid of small cross-section in which there are two coils C_1 and C_2 of insulated wires, one wound over the other. Let N_1 and N_2 be the number of turns in C_1 and C_2 respectively, Fig.4.14.

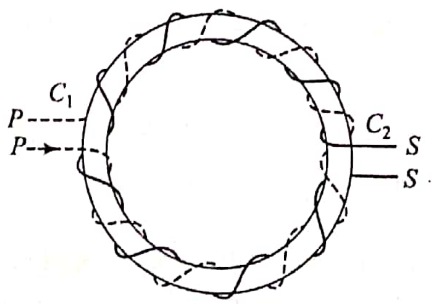


Fig. 4.14

If I_1 be the current in C_1, magnetic field produced within the toroid is

$$B = \frac{\mu_0}{2\pi}\frac{N_1 I_1}{r}$$

Here r is the mean radius of toroid. The field is assumed to be constant.

$\therefore$ Flux through C_1 is

$$\Phi_{11} = BN_1 A = \frac{\mu_0 N_1^2 I_1 A}{2\pi r} = L_1 I_1$$

Flux through C_2 is

$$\Phi_{12} = BN_2 A = \frac{\mu_0 N_1 N_2 I_1 A}{2\pi r} = M_{12} I_1$$

$$\therefore L_1 = \frac{\mu_0 N_1^2 A}{2\pi r} \text{ and } M_{12} = \frac{\mu_0 N_1 N_2 A}{2\pi r} \text{ ....(i)}$$

If I_2 be the current in coil C_2 then by exactly similar calculation we shall arrive at the relations

$$L_2 = \frac{\mu_0 N_2^2 A}{2\pi r} \text{ and } M_{21} = \frac{\mu_0 N_1 N_2 A}{2\pi r} \text{ ...... (ii)}$$

Here by definitions L_1 and L_2 are the self-inductances of the coils. M_{12} and M_{21} are the mutual inductances representing the effect of C_1 on C_2 and C_2 on C_1. respectively. We find

$$M_{12} = M_{21}$$

So mutual inductance between two coils is represented by M.

From relation (i) and (ii) we find that

$$L_1 L_2 = M^2 \text{ or, } M = \sqrt{L_1 L_2}$$

We get the relation between self-inductance and mutual inductance in the *very special condition* that *all the flux* produced in one coil *is linked* with the other. This is the maximum magnetic coupling that is possible between two coils. Hence we can conclude that the maximum value of mutual inductance of two coils having self-inductances L_1 and L_2 is given by

$$M_{max} = \sqrt{L_1 L_2}$$

In fact, there is always some leakage of flux and therefore, $M < \sqrt{L_1 L_2}$.

By definition, *coefficient of coupling* of two magnetically coupled coils, $k = \frac{M}{\sqrt{L_1 L_2}}$.

We see that if the coefficient of coupling $k = 1$, *all the flux produced by one coil is linked by the other*. It is also called tight coupling. It is an ideal condition.

If $k < 1$, it is called loose coupling; there is leakage of flux.

If there is no common flux between two coils, they are said to be magnetically isolated. In this case $M = 0$, $k = 0$.

4.3.3 *Energy stored in two magnetically coupled coils*

We consider two coils C_1 and C_2, which have self-inductances L_1, L_2 and mutual inductance M. Currents through them are I_1 and I_2, Fig.4.15. We like to find the magnetic energy stored in this system. We have to find how much work has to be done to establish these currents against the emfs induced by self and mutual induction.

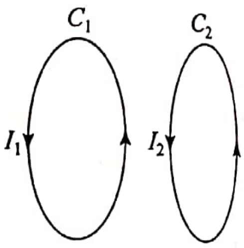


Fig. 4.15

Let i_1 and i_2 be the currents in the two coils C_1 and C_2 at an instant during the growths of current from 0 to I_1 and from 0 to I_2.

The emfs induced in the two coils at that instant are

$$\epsilon_1 = -L_1 \frac{di_1}{dt} \mp M \frac{di_2}{dt}$$

$$\text{and } \epsilon_2 = -L_2 \frac{di_2}{dt} \mp M \frac{di_1}{dt}$$

The sign $\mp$ before the second terms is necessary, because coupling of two coils may be of two kinds. If the currents in the two coils are flowing in the *same* direction, increase of flux in one coil causes increase of flux in the other. This is called *positive* coupling and *negative* sign should occur in the above equations. If currents, however, are in *opposite* directions, then increase of current in one causes decrease of flux in the other. This is called *negative* coupling and *positive* sign should occur in the above equations.

In time dt charges flown in the two coils are respectively $i_1 dt$ and $i_2 dt$.

Work done against the above emfs in time dt in the two coils are

$$dW_1 = \epsilon_1 i_1 dt = L_1 \frac{di_1}{dt} i_1 dt \pm M \frac{di_2}{dt} i_1 dt$$

$$dW_2 = \epsilon_2 i_2 dt = L_2 \frac{di_2}{dt} i_2 dt \pm M \frac{di_1}{dt} i_2 dt$$

$dW = dW_1 + dW_2 = L_1 i_1 di_1 \pm M di_2 i_1 + L_2 i_2 di_2 \pm M i_2 di_1$

$= L_1 i_1 di_1 \pm M d(i_1 i_2) + L_2 i_2 di_2$

To get the total work done to establish currents I_1 and I_2 in the two coils we have to integrate dW from 0 to I_1 and from 0 to I_2. We get

$$W = \frac{1}{2}L_1 I_1^2 \pm M I_1 I_2 + \frac{1}{2}L_2 I_2^2$$

This is the energy stored up as magnetic field energy in the system of two coils.

∴ Energy of the system,

$$U_m = \frac{1}{2}L_1 I_1^2 \pm M I_1 I_2 + \frac{1}{2}L_2 I_2^2 \quad ....(4.18)$$

The first and the third term are the energies due to self induction. These are called the self energies and these arise from the interaction of each coil with its own field. The second term is interaction arising from mutual inductance. We can also express the energy as

$$U_m = \frac{1}{2}I_1(L_1 I_1 \pm M I_2) + \frac{1}{2}I_2(L_2 I_2 \pm M I_1)$$

$$= \frac{1}{2}I_1 \Phi_{m1} + \frac{1}{2}I_2 \Phi_{m2}$$

Here Φ_{m_1} and Φ_{m_2} are the total magnetic fluxes linking C_1 and C_2 respectively.

4.3.4 *Mutual Inductance of two coils*

1. Coaxial coils : In Fig.4.16 we can see two *coaxial* solenoids S and P. The shorter one S is wound over P, the longer one. Let n_1 be the number of turns per unit length, A be the area of cross-section of the coil P. If current through P is I, the magnetic field near the middle if the coil P is

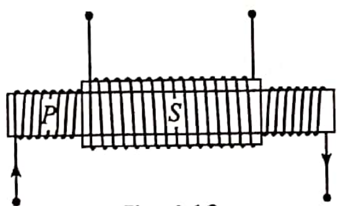


Fig. 4.16

$$B = \mu_0 n_1 I$$

If we suppose that there is no leakage of flux and total number of turns of the coil S be N_2 then the flux linked with the coil S is

$$\Phi_m = N_2 BA = N_2 \mu_0 n_1 I A$$

Putting $I = 1$, we get the mutual inductance between the two coils given by

$$M = \mu_0 N_2 n_1 A$$

If the coil S is within the longer coil P, then A in the above formula should be the area of coil S.

2. Two coaxial parallel circular coils :

In Fig. 4.17 we see identical parallel coils C_1 and C_2, each of radius a and each having a turns placed at a distance x from each other. If I be the current passing through C_1, then the magnetic field at O_2 is

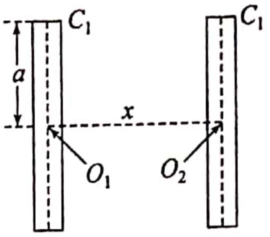


Fig. 4.17

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{n 2 I \pi a^2}{(a^2 + r^2)^{\frac{3}{2}}}\hat{r}$$

If we suppose that the field is uniform over the cross-section of the coil, the magnetic flux linked with C_2 is

$$\Phi_m = B \cdot n\pi a^2 = \frac{\mu_0}{4\pi}\frac{n 2 I \pi a^2}{(a^2 + r^2)^{\frac{3}{2}}} \times n\pi a^2$$

$$= \frac{\mu_0}{4\pi}\frac{2\pi^2 a^4 n^2 I}{(a^2 + r^2)^{\frac{3}{2}}}$$

Putting $I = 1$, we get the mutual inductance,

$$M = \frac{\mu_0}{4\pi}\frac{2\pi^2 a^4 n^2}{(a^2 + r^2)^{\frac{3}{2}}}.$$

From symmetry we shall get the same flux linked with the coil C_1.

∴ Mutual inductance between the two coils is

$$M = \frac{\mu_0}{4\pi}\frac{2\pi^2 a^4 n^2}{(a^2 + r^2)^{\frac{3}{2}}}.$$

4.5 Inductors in series and in parallel

In many electric circuits we require to add self inductance for different purposes. We use conducting wire wound in the form of a coil. Such coils have generally negligible resistances. These coils are called inductors. As capacitors can store electric energy, inductors store magnetic energy. These capacitors and inductors have very important roles to play in alternating current circuits.

Inductors may be joined in series and in parallel just like resistors and capacitors. We now like to find equivalent inductances of such combinations of inductors.

Inductors in series: In Fig.4.18 we see two inductors of inductances L_1 and L_2 and having

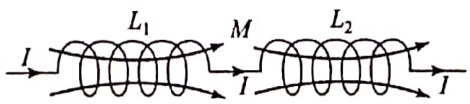


Fig. 4.18

negligible resistances, joined in series. We suppose that the two coils are *far apart* so that there is no mutual inductance.

If current through the coils at a particular instant is I and it is changing with time, emf induced in the two coils are

$$\in = -L_1\frac{dI}{dt} - L_2\frac{dI}{dt} \quad \text{.....................(i)}$$

If the same emf is induced in a single coil for the same variation of current, then inductance (L) of that coil is the equivalent inductance.

$$\therefore\ -L\frac{dI}{dt} = -L_1\frac{dI}{dt} - L_2\frac{dI}{dt} \quad \therefore L = L_1 + L_2.$$

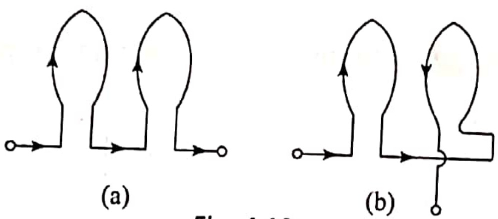
Now suppose that the two coils are near to each other and the mutual inductance cannot be neglected. Suppose the mutual inductance of the two coils is M. Then the above eqn.(i) becomes

$$\in = -L_1\frac{dI}{dt} - M\frac{dI}{dt} - L_2\frac{dI}{dt} - M\frac{dI}{dt} \quad \text{..........(ii)}$$

By the same reasoning as above we get the equivalent or total inductance given by

$$L = L_1 + L_2 + 2M$$

In eqn. (ii) we have assumed that the two emf developed by mutual inductances are in the same direction as those by self induction. This happens when the currents in the two coils are in the same direction and so, self and mutual fluxes reinforce, Fig.4.19a.



(a) (b)

Fig. 4.19

But in the opposite case, when the currents in the two coils are in opposite directions, Fig.4.19b and so the self and mutual fluxes are in opposition, we shall get

$$L = L_1 + L_2 - 2M$$

$\therefore$ General relation for series combination is

$$L = L_1 + L_2 \pm 2M \quad \text{.................(4.19)}$$

Inductors in parallel : In Fig.4.20, we see two inductors of inductances L_1 and L_2 joined in parallel and mutual inductance between them is M.

Let instantaneous currents in the two coils be I_1 and I_2 and they vary. We suppose that currents in the two coils are in the same direction and so, self and mutual fluxes reinforce. Therefore e[mf] induced in the two coils are

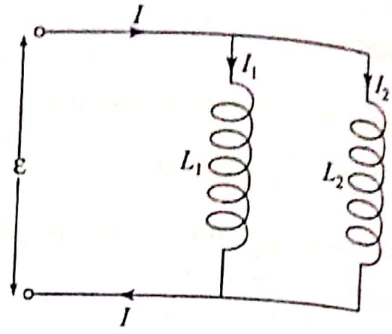


Fig. 4.20

$$\in_1 = -L_1\frac{dI_1}{dt} - M\frac{dI_2}{dt}$$

and $$\in_2 = -L_2\frac{dI_2}{dt} - M\frac{dI_1}{dt}$$

As the coils are joined in parallel $\in_1 = \in_2 = \in$ and total current, $I = I_1 + I_2$.

Solving the above two equations by the method of cross-multiplication, we get

$$\frac{dI_1}{dt} = -\frac{\in(L_2 - M)}{L_1L_2 - M^2} \text{ and } \frac{dI_1}{dt} = -\frac{\in(L_1 - M)}{L_1L_2 - M^2}$$

Now $$\frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt} = -\frac{\in(L_1 + L_2 - 2M)}{L_1L_2 - M^2}$$

$$\therefore \in = -\frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}\frac{dI}{dt} \quad \text{...............(i)}$$

If L is the equivalent inductance of the combination, we have $\in = -L\frac{dI}{dt}$ ....(ii)

Comparing (i) and (ii) we get the equivalent inductance is

$$L = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}$$

If the self and mutual fluxes are in opposition, we have

$$L = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M}$$

General equation for parallel combination is

$$L = \frac{L_1L_2 - M^2}{L_1 + L_2 \mp 2M} \quad \text{................(4.20)}$$

If the mutual inductance may be neglected,

$$L = \frac{L_1L_2}{L_1 + L_2}.$$

4.6 Energy stored in magnetic field

The energy of the magnetic field within a coil of self-inductance L carrying current I is given by

eqn. $$U_m = \frac{1}{2}LI^2 \quad \text{...................(i)}$$

We consider a long solenoid and take a length l of it at the middle. If the area of cross-section is A, the volume of this portion within the solenoid is Al. When current flows through it, magnetic field within this portion is quite uniform. Also the magnetic field is almost entirely contained within this volume, as magnetic field outside a long solenoid is essentially zero. The magnetic field is given by

$$B = \mu_0 nI \quad \text{.......................(ii)}$$

where I is current and n is the number of turns per unit length of the coil.

The self-inductance of this portion of the solenoid is given by eqn.

$$L = \mu_0 n^2 lA \quad \text{....................(iii)}$$

Substituting the values of L and I from (ii) and (iii) in (i) we get

$$U_m = \frac{1}{2} \times \mu_0 n^2 lA \times \left(\frac{B}{\mu_0 n}\right)^2 = \frac{1}{2}\frac{B^2}{\mu_0} lA$$

This is the energy contained in volume lA within the solenoid. Therefore, magnetic energy per unit volume or energy density is

$$u_m = \frac{U_m}{lA} = \frac{1}{2}\frac{B^2}{\mu_0} \quad \text{......................(4.21)}$$

We have calculated it for the special case of a solenoid, but it is true for all magnetic configurations.

We did similar calculation for electric energy density for the special case of a parallel-plate capacitor.