ASUTOSH COLLEGE

4th Semester

1. Subject Name: Mathematics (Honours)
2. Semester – 4th
3. Name of Teacher: Dr. N. Ghosh
4. Topic: Uniform Convergence of Sequence and Series of functions:

HOME TASK

Please choose the Correct option with proper justification:

1) Let $f_n(x) = x^n$, $x \in [0,1]$. Then

a) $\{f_n\}$ is not Convergent Pointwise in $[0,1]$.

b) $\{f_n\}$ is Convergent Pointwise but not Uniformly on $[0,1]$.

c) $\{f_n\}$ is Uniformly Convt. on $[0,1]$.

d) $\{f_n\}$ Converge at $x=0$ only.

2) Let $f_n(x) = \dfrac{\sin nx}{\sqrt{n}}$, $x \in [-1,1]$. Then

a) $\{f_n\}$ does not converge Uniformly on $[-1,1]$.

b) $\lim\limits_{n\to\infty} \int_{-1}^{1} f_n(x)\,dx \neq 0$

c) $\{f_n'\}$ does not converge Uniformly on $[-1,1]$.

d) $\{f_n\}$ Converges Uniformly on $[-1,1]$.

3) If $y_n = \int_1^2 \dfrac{e^{-nt}}{t}\,dt$, then

a) $\{y_n\}$ is Convergent to 0.

b) $\{y_n\}$ is Convergent to 1.

c) $\{y_n\}$ is not Convergent.

d) $\{y_n\}$ is diverge to $+\infty$.

4) Pick out the largest of the sets given below on which the sequence of functions $\{e^{-n\cos^2 x}\}$ converges uniformly

a) $[0, \frac{9\pi}{20}) \cup (\frac{11\pi}{20}, \pi]$

b) $[0, \pi/2) \cup (\pi/2, \pi]$

c) $[0, \frac{\pi}{2}-\delta) \cup (\pi/2+\delta, \pi]$, $0 < \delta < \frac{\pi}{100}$

d) $[0, \pi]$

5.) The series $\sum_{n=1}^{\infty} \frac{x}{n(1+nx)^2}$ is

a) convergent on $[0,1]$ but not uniformly

b) divergent on $[0,1]$.

c) Uniformly convergent on $[0,1]$

d) none of these.

6) The series $\sum_{n=1}^{\infty} \frac{\sin nx}{n^2}$ is

a) uniformly convergent

b) only point-wise convergent.

c) divergent

d) none of these.

7) Let $\{f_n\}$ be a sequence of monotone increasing real-valued functions on $[0,1]$ such that $\{f_n\}$ converges pointwise to the function $f \equiv 0$ on $[0,1]$. Then

a) $\{f_n\}$ converge to f uniformly.

b) $\{f_n\}$ is not uniformly convergent.

c) If $f_n(x) \geq 0$ $\forall n$, $\forall x$ then $f_n(x)$ must be continuous for sufficiently large n.

d) $\{f_n\}$ converges uniformly only when each f_n is continuous.