chapter 14

The Demand for Money

The demand for money was introduced in Chapter 4 as the demand for real money balances, $m = M/P$. There we suggested, in a fairly loose way, that the demand for real balances could be divided into a *speculative* demand component, inversely related to the interest rate, and a *transactions* demand component, positively related to income and inversely related to the interest rate. This gave us the demand-for-money function in Part II:

$$\frac{M}{P} = m = m(r, y) \approx l(r) + k(y), \qquad (1)$$

where $\partial m/\partial r$ is negative and $\partial m/\partial y$ is positive.

Since the 1930s, economists have developed the theory underlying the demand for money along several different lines, each of which provides a different answer to the basic question: If bonds earn interest and money doesn't, why should a person hold money? While the way the various theories answer this question differs, in general they come down to a demand-for-money function similar to the one shown in equation (1).

In this chapter we will develop four prominent approaches to the demand for money. The first is the *regressive expectations* model attributed to Keynes and described by Tobin in his article on liquidity preference. This model essentially says that people hold money when they expect bond prices to fall, that is, interest rates to rise, and thus expect that they would take a loss if they were to hold bonds. Since people's estimates of whether the interest rate is likely to rise or fall, and by how much, vary fairly widely, at any given

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interest rate there will be someone expecting it to rise, and thus someone holding money.

The obvious problem with this view is that it suggests that individuals should, at any given time, hold *all* their liquid assets either in money or in bonds, but not some of each. This is obviously not true in reality. The second approach, Tobin's model of liquidity preference, deals with this problem by showing that if the return on bonds is uncertain, that is, bonds are risky, then the investor worrying about both risk and return is likely to do best by holding both bonds and money.

A third approach to the demand for money is the inventory approach to transactions demand developed by both Baumol and Tobin. They show that there is a transactions need for money to smooth out the difference between income and expenditure streams, and that the higher the interest rate—the return on holding bonds instead of money—the smaller these transactions demand balances should be. Finally, we will look at Friedman's modern version of the quantity theory of money. Friedman analyzes the demand for money as an ordinary commodity. It can be viewed as a producer's good; businesses hold cash balances to improve efficiency in their financial transactions and are willing to pay, in terms of foregone interest income, for this efficiency. Money can also be viewed as a consumer's good; it yields utility to the consumer in terms of smoothing out timing differences between the expenditure and income streams and also in terms of reducing risk. This type of analysis brings Friedman to much the same demand-for-money function as that based on the other theories.

Our discussion of the demand for money initially focuses on the individual's decision concerning the composition of her liquid assets. We assume that she has a given amount of liquid wealth W which remains unchanged during the period under discussion. She must decide how much of that liquid wealth should be allocated to each of two kinds of assets: money (M), defined as currency plus demand deposits, which is riskless and does not earn interest; and bonds (B), which do earn interest and bear a liquidity risk. This is the risk that they might have to be sold at a capital loss if, when money is needed, bond prices are lower than they were when the bonds were purchased. Later in our analysis, we will see how the individual's preferences can be generalized into a community liquidity preference. We can begin with the regressive expectations model of the demand for money.

THE REGRESSIVE EXPECTATIONS MODEL

Our development of the regressive expectations model follows Tobin's analysis in his article on liquidity preference. A bond holder has an expected return

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on the bond from two sources: the bond's yield—the interest payment he receives; and a potential capital gain—an increase in the price of the bond from the time he buys it to the time he sells it. The bond's yield Y is usually stated as a percentage of the face value of the bond. The market rate of return on the bond r is the ratio of the yield to the price of the bond P_b. For example, if a \$100 bond has a yield of \$5, the percentage yield is 5 percent. If the price of the bond rises to \$125, the \$5 yield corresponds to a market rate r of 4 percent—\$5/\$125. Thus, the market rate is given by

$$r = \frac{Y}{P_b}, \tag{2}$$

and, since the yield Y is a fixed amount stated as a percentage of the bond's face value, the market price of a bond is given by

$$P_b = \frac{Y}{r}. \tag{3}$$

The expected percentage capital gain g is the percentage increase in price from the purchase price P_b to the expected sale price P_b^e. This gives us an expression for the percentage capital gain, $g = (P_b^e - P_b)/P_b$. From equations (2) and (3), with a fixed Y on the bond, an expected price P_b^e corresponds to an expected interest rate, $r^e = Y/P_b^e$. Thus, in terms of expected and current interest rates, the capital gain can be written as

$$g = \frac{Y/r^e - Y/r}{Y/r}.$$

Canceling the Y terms and multiplying the numerator and denominator by r gives us

$$g = \frac{r}{r^e} - 1 \tag{4}$$

as the expression for expected capital gain in terms of current and expected interest rates. For example, if the present market interest rate is 5 percent, and the purchaser of the bond expects the rate to drop to 4 percent, his expected capital gain will be

$$g = \frac{0.05}{0.04} - 1 = 1.25 - 1 = 0.25, \text{ or 25 percent.}$$

The total rate of return on a bond—e for earnings—will be the sum of the market rate of interest at the time of purchase and the capital gains term.

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Thus, $e = r + g$, and substituting for g from equation (4), we have an expression for the total rate of return

$$e = r + \frac{r}{r^e} - 1. \tag{5}$$

The Individual's Demand-for-Money Function

Now with an expected return on bonds given by e, and with a zero return on money, the asset holder can be expected to put his liquid wealth into bonds if he expects the return e to be greater than zero. If the return on bonds is expected to be less than zero, he will put his liquid wealth into money.

In the regressive expectations model, each person is assumed to have an *expected* interest rate r^e corresponding to some *normal* long-run average rate. If rates rise above this long-run expectation, she expects them to fall, and vice versa. Thus, her expectations are *regressive.* Initially, we will assume that her expected long-run rate doesn't change much with changes in current market conditions.

The asset holder's expected interest rate r^e, together with the observable market interest rate r, determines her expected percentage return e. Given this, we can compute the critical level of the market rate r, r_c, which would give her a net zero return on bonds, that is, the value of r that makes $e = 0$. When actual $r > r_c$, we would expect her to hold all of her liquid wealth in bonds. When $r < r_c$, she moves 100 percent into money. To find this critical value of r, r_c, we set the total return shown in equation (5) equal to zero:

$$0 = r + \frac{r}{r^e} - 1;$$

$$r(1 + r^e) = r^e;$$

and thus

$$r = \frac{r^e}{1 + r^e} = r_c. \tag{6}$$

Here r_c, the value of the market interest rate r that makes $e = 0$, is given by $r^e/(1 + r^e)$.

This relationship between the individual's demand for real balances and the interest rate is shown in Figure 14-1. Here we label the horizontal axis to show the demand for real balances, since later developments will show, as suggested in Chapter 4, that it is the demand for real balances, $m = M/P$, that depends on the interest rate. Since we are implicitly holding the general price level constant throughout this section, changes in real balances M/P correspond

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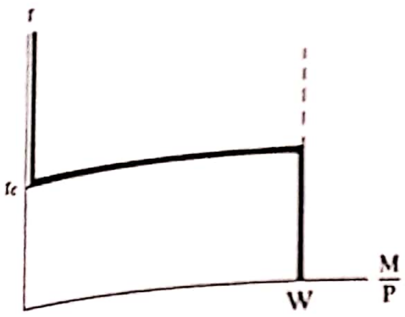

Figure 14-1 Individual's demand for money in the no-risk case.

to changes in M. Hence, the picture in Figure 14-1 will be the same whether we label the axis M or M/P.

In Figure 14-1, when r is greater than r_c, the asset holder puts all of W into bonds, so that her demand for money is zero. As r drops below r_c so that $e < 0$ and expected capital losses on bonds outweigh the interest yield, the asset holder moves her entire liquid wealth into money. This gives us a demand-for-money curve for an individual that looks like a step function. When r exactly equals r_c, $e = 0$ and the asset holder is indifferent between bonds and money. At any other value of r, the asset holder is either 100 percent in money or 100 percent in bonds.

Up to now, we have assumed that the individual has a *given* expected interest rate r^e that is not sensitive to changes in the market rate r. What would happen if the expected interest rate r^e depended positively on the current interest rate? Might an increase in present r raise r^e enough to raise r_c, thus *increasing* the demand for money?

Suppose $r^e = f(r)$, $f' > 0$, so that from expression (6) for r_c we have

$$r_c = \frac{f(r)}{1 + f(r)} = h(r). \tag{7}$$

Then if $h' < 1$, we could plot r_c against r as shown in Figure 14-2, where the $h(r)$ curve has a slope less than unity. Where $h(r)$ crosses the 45° line, at $r = r_0$, r also is equal to r_c, and the person is indifferent between bonds and

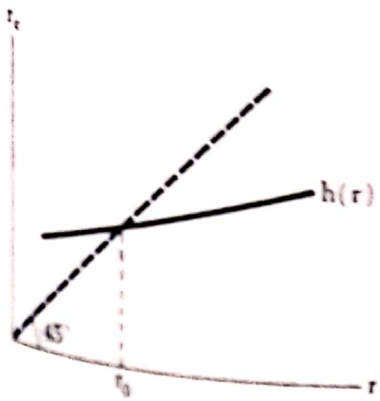

Figure 14-2 Critical interest rate, r_c, versus the actual interest rate, r.

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money; she is on the horizontal segment of the demand curve of Figure 14-1. When r is greater than r_0, r is also greater than r_c along $h(r)$, and the person puts her entire liquid wealth into bonds. If r is less than r_0 in Figure 14-2, r is also less than r_c, and the individual is 100 percent in money. Thus, if $h' < 1$, the individual's demand-for-money function will look like that of Figure 14-1, even with $r^e = f(r)$.

What are the conditions under which $h' < 1$, that is, $dr_c/dr < 1$? Differentiating equation (7) with respect to r, we have

$$\frac{dr_c}{dr} = \frac{f'(r)}{1 + f(r)} - \frac{f(r)f'(r)}{[1 + f(r)]^2}.$$

Simplifying this gives us

$$\frac{dr_c}{dr} = \frac{[1 + f(r)]f'(r) - f(r)f'(r)}{[1 + f(r)]^2}$$

and

$$h'(r) = \frac{dr_c}{dr} = \frac{f'(r)}{(1 + r^e)^2}. \tag{8}$$

Since r^e is presumably positive, so that the denominator of (8) is greater than one, a sufficient condition for $h' < 1$ is that $f'(r) \equiv dr^e/dr$ is less than one. Thus, the individual's money demand function will resemble that of Figure 14-1 if $f'(r) < 1$, that is, if an increase in the current market rate r, raises r^e by less than the increase in r. Expectations must be sufficiently *regressive* that movements in the current market rate cause smaller movements in expected rates. If this is the case, the Figure 14-1 demand-for-money curve will hold in this no-risk model where expectations concerning r^e are held with certainty.

The Aggregate Demand-for-Money Function with Regressive Expectations

The individual demand curves can be aggregated for the entire money market as follows. Locate the individual with the highest critical interest rate, r_c^{max} in Figure 14-3. As the interest rate falls below that r_c^{max}, she shifts all of her liquid wealth into money. As the interest rate drops, more individual r_c's are passed and more people shift from bonds to money. Eventually, r will drop far enough that no one will want to put liquid wealth into bonds, and the demand for money will equal total liquid wealth, ΣW.

Figure 14-4 shows the frequency distribution of the critical interest rates. The area under a frequency distribution equals 100 percent, and for any level of r_c, the area under the curve to the left of that r_c gives the proportion of people with r_c less than that r_c. The population average r_c is shown as $\bar{r}_c$ in

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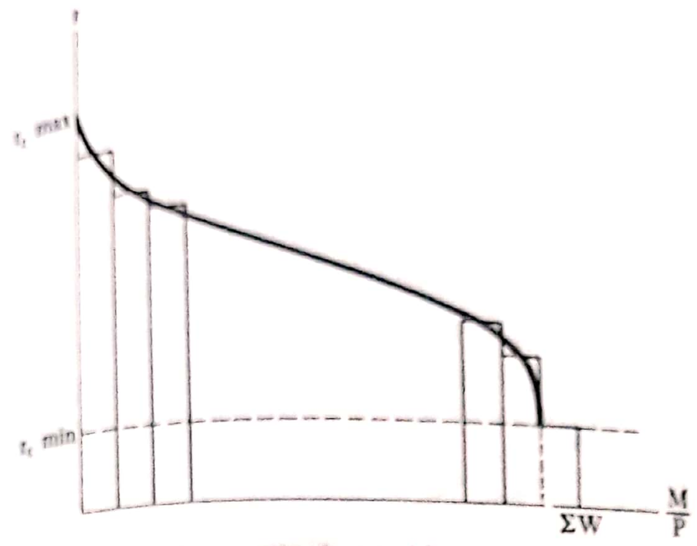

Figure 14-3 Aggregate demand for money in the no-risk case.

Figure 14-4. If r_c's are distributed among the population as shown in Figure 14-4, that is, few people having extreme r_c's and more people bunched around a central $\bar{r}_c$, then the aggregate demand-for-money curve will have the shape shown in Figure 14-3—more steeply sloped at the ends than in the middle for a given aggregate liquid wealth, ΣW.

Aggregate Wealth and the Interest Rate

It should be noted that the actual demand-for-money function would be flatter than the curve shown in Figure 14-3, which assumes ΣW is independent of r. In fact, as interest rates go down, aggregate wealth increases, owing to an increase in bond prices through equation (3). Thus, we would observe for each drop in the interest rate a slightly greater shift into money than that shown in Figure 14-3 because each individual would have more "bond wealth" to change into "money wealth" as the rate falls. Each increase in the demand for money is due to a *wealth effect* as well as to an *interest rate effect*.

This point is illustrated in Figure 14-5. The curves d_0, d_1, d_2 are each drawn on the assumption of a *given* aggregate wealth ΣW, shown along the horizontal axis. Movement along any one demand curve shows the effect of

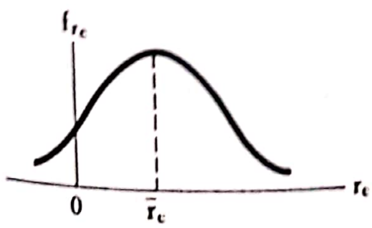

Figure 14-4 Frequency distribution of the critical interest rates.

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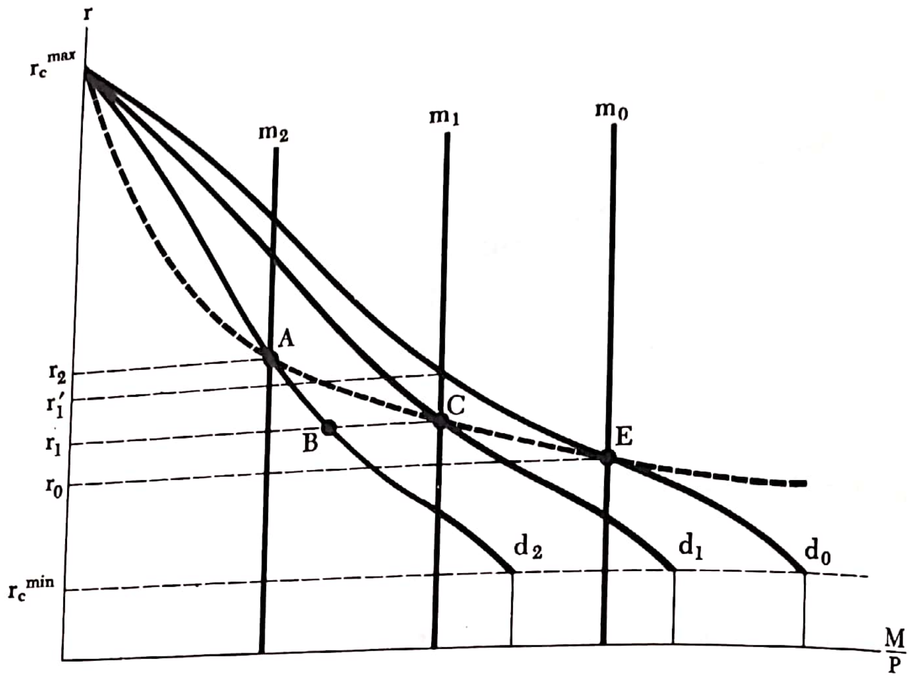

Figure 14-5 The wealth effect on the demand for money.

changes in the interest rate on the demand for money *directly,* but leaves out the effect through a change in ΣW. As interest rates fall, not only is there a shift of a given liquid wealth from bonds to money, but the amount of liquid wealth in the economy increases. Hence, there is a *further* expansion in the demand for money through a shift in the demand curve.

Suppose that at interest rate r_2 the demand for money is at point *A.* All people with a critical interest rate above r_2 hold money, and all people with critical rates below r_2 hold bonds, giving a demand for money at m_2. Suppose, now, that the interest rate drops to r_1. There is an increase in the price, or value, of bonds, since that value is inversely related to the level of interest rate, as shown in equation (3). Thus, the demand for money increases, *not* to *B* on the old demand curve d_2 but to *C,* on the new demand curve d_1.

We can now see how the true demand-for-money function is traced by following the effects of changes in the money supply. We have seen that the demand for money depends on the level of interest rate. Suppose there is an initial equilibrium point *E* where demand for money equals supply, and the interest rate is r_0. Now the government decides to reduce the supply of money to m_1 through action by the Federal Reserve Board. We know that this will raise the interest rate. On the old demand curve d_0 the interest rate would settle at r_1'. However, because interest rates are increasing, there is a *shift* in the demand curve to d_1. Interest rates rise from r_0 and at the same time demand shifts until a new equilibrium interest rate is reached at r_1, which equates demand and supply. This gives us a new equilibrium point, *C.* Should there be a further reduction in the money supply, the same process would be

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repeated until a new equilibrium level of r_c were reached at A. A reversal of the process, whereby the money supply shifted out from m_2 to m_1 to m_0, would give the same equilibrium points A, C, and E. Thus, by connecting all such equilibrium points, the demand-for-money curve which incorporates both interest rate and wealth effects can be described.

Figure 14-5 should make it clear that as the interest rate drops, there are continuing increments to liquid wealth, so that as the interest rate approaches zero, liquid wealth approaches infinity. In other words, the demand curve becomes flatter and flatter as r decreases, eventually approaching the minimum r_c value asymptotically in this static expectations model.

Thus, the regressive expectations model yields a demand-for-money function that looks much like the one we have been using so far in this book. As interest rates fall, the demand for money increases, and the demand curve is likely to be convex. That is, successive interest rate decreases of equal amounts will bring increasing increments in the demand for money.

There are two troublesome aspects of this analysis, however. In the first place, if the money market remained in equilibrium for a long enough period, people should begin to adjust their expected interest rates to correspond to the actual prevailing interest rate. They would all tend to adopt eventually the same critical interest rate as time passes, so that the aggregate demand curve for the entire money market would increasingly look like the flat curve of Figure 14-1, instead of the negatively sloped demand curve with a variety of critical rates shown in Figure 14-3. This implication of the regressive expectations model—that the elasticity of demand for money with respect to changes in the interest rate is increasing over time—is not supported by empirical studies.

Second, if we assume that people actually do have a critical interest rate as shown in Figure 14-2, then the clear implication of the model is that, in this two-asset world, individuals hold either all bonds or all money, never a mix of the two. The negative slope of the aggregate demand curve is due to the fact that people disagree about the value of r^e, and thus in their critical rates r_c. In fact, however, individuals do not hold portfolios consisting of just one asset. In general, portfolios hold a mixture of assets: they are *diversified.* An explanation of this result—that people hold both money and bonds at the same time—can be found in the portfolio balance approach to the demand for money developed by Tobin.

THE PORTFOLIO BALANCE APPROACH

The portfolio balance approach begins with the same expression for total percentage return e that we developed in the last section,

$$e = r + g. \tag{9}$$

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In that section we assumed that the percentage rate of expected capital gain, given by

$$g = \frac{r}{r^e} - 1, \tag{10}$$

is determined with certainty by the individual; she chooses r^e as a function of r, and no consideration of uncertainty, or risk, enters the problem. The basic contribution of the portfolio balance approach is to enter risk considerations explicitly into the determination of the demand for money.

The Probability Distribution of Capital Gains

Rather than some *fixed* expected capital gain, here we will assume that the asset holder has a whole spectrum of expected capital gains, each with a probability of its occurrence attached. Such a *probability*, or *frequency*, *distribution* of expected gains is shown in Figure 14-6. Each possible value of capital gain g has a probability f_g attached to it. If one asks the asset holder what the probability is of achieving a gain greater than $\bar{g}$, say, 15 percent, his answer will be the area under the probability distribution to the right of $\bar{g}$. Thus, the asset holder is not certain of the value of g he expects, but has an implicit distribution of these gains around some central value—the average, or *expected* gain, $\bar{g}$.

If the probabilities of capital gains are distributed "normally"—according to the familiar symmetrical bell-shaped distribution shown in Figure 14-6—then we have a natural measure of uncertainty, or risk. This measure is the *standard deviation*, σ_g, of the probability distribution of capital gains. To find the standard deviation of expected gains, σ_g, we can locate the two points symmetrically opposite each other on the normal probability distribution that have the following property: The area under the curve between these two points is two-thirds of the total area under the curve. Given the shape of the normal curve, these points are also the inflection points of the curve, where it turns from concave to convex. The standard deviation of the probability distribution, σ_g, is the distance between either of these two points and the mean of the distribution, $\bar{g}$. The statistical significance of σ_g is that, since two-thirds of the area under the curve is between the points $\bar{g} - \sigma_g$ and $\bar{g} + \sigma_g$, the

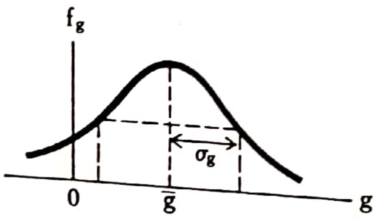

Figure 14-6 Probability distribution of expected gains.

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asset holder has a 66.7 percent chance that the *actual* g will turn out between $\bar{g} \pm \sigma_g$. Thus, if $\bar{g} = 10$ percent, and $\sigma_g = 2$ percent, the investor has a two-thirds chance that actual g will be between 8 and 12 percent.

That the standard deviation is a natural measure of the riskiness of bonds can be seen by considering the two probability distributions, both with the same $\bar{g}$, shown in Figure 14-7. The narrow distribution, f_1, illustrates a case in which the asset holder is very *certain* of the gain—it has a small σ_g. The wider distribution f_2 shows a case in which, with the same central expected gain $\bar{g}$, the investor has a very uncertain estimate of the gain; thus, σ_{g2} is greater than σ_{g1}. If we can identify riskiness with uncertainty, σ_g is a measure of the risk of holding liquid assets in bonds.

Now in place of a return expected with certainty, e, we have an expected return, $\bar{e}$, where

$$\bar{e} = r + \bar{g}, \tag{11}$$

and $\bar{g}$ is the mean expected capital gain from the probability distribution of Figure 14-6. If the asset holder is putting B dollars of her liquid assets into bonds, her expected total return $\bar{R}_T$ is then

$$\bar{R}_T = B \cdot \bar{e} = B \cdot (r + \bar{g}). \tag{12}$$

Similarly, if the standard deviation of return on a bond is σ_g, a number like 2 percent, and all bonds are alike, then the total standard deviation of bond holdings is given by

$$\sigma_T = B \cdot \sigma_g. \tag{13}$$

The Individual's Portfolio Decision

Equations (12) and (13) give us the technical situation facing the asset holder—the budget constraint along which he or she can trade increased risk σ_T for increased expected return $\bar{R}_T$. They also give the investor a formula for deciding

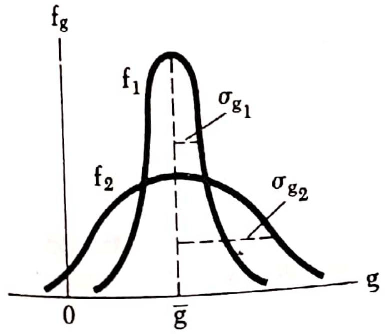

Figure 14-7 Different risks for bonds with the same expected gain.

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how much funds to put into bonds to achieve a given risk-return mix along the budget line. From (13) we have

$$B = \frac{\sigma_T}{\sigma_g} = \frac{1}{\sigma_g}\sigma_T. \tag{14}$$

With σ_g fixed by the asset holder's probability distribution, (14) gives the total bond holdings B needed to attain any given level of risk σ_T. Using this expression to replace B in (12) gives us the budget constraint,

$$\bar{R}_T = \frac{\sigma_T}{\sigma_g}(r + \bar{g}) = \sigma_T\left(\frac{r + \bar{g}}{\sigma_g}\right). \tag{15}$$

Here r is a known current value, fixed, at least to the individual, by the bond market. The investor knows $\bar{g}$ and σ_g, at least implicitly, from the probability distribution of g's in Figure 14-6. Thus, the expression in parentheses in (15) is a given, determined number which gives the constant rate of trade-off between return $\bar{R}_T$ and risk σ_T. Differentiating (15) we have

$$\frac{d\bar{R}_T}{d\sigma_T} = \frac{r + \bar{g}}{\sigma_g}. \tag{16}$$

If r is, say, 5 percent, $\bar{g}$ is 10 percent, and σ_g is 5 percent, then $dR_T/d\sigma_T$ will be 3. In this case, an increase of one percentage point in the standard deviation in the total portfolio σ_T will buy a 3 percent increase in expected total return $\bar{R}_T$.

The budget constraint (15) for an individual asset holder is shown in the top half of Figure 14-8. The standard deviation of the total portfolio, σ_T, is shown on the horizontal axis. The vertical axis above the horizontal axis measures the expected rate of return on the portfolio, $\bar{R}_T$. The straight line in the top half shows the trade-off between risk and expected return which faces the individual. Its slope, derived from equation (16), is $(r + \bar{g})/\sigma_g$. Each of these terms, and thus the slope of the budget line, is fixed for each individual.

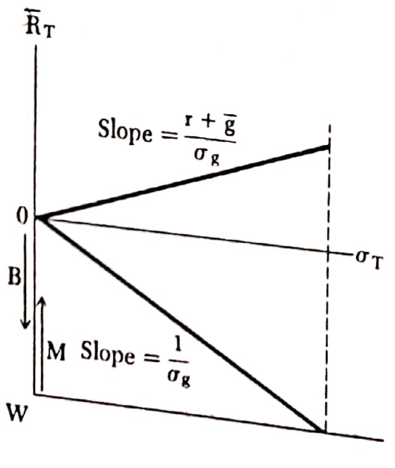

Figure 14-8 Budget constraint for individual trading increased risk for increased return.

The length of the vertical axis below the horizontal axis in Figure 14-8 is given by the total liquid wealth W of the individual. The distance from the origin along this axis gives total bond holdings B; the distance from W $(=W-B)$ gives money holdings M. For any given value of σ_T we can locate the value of B by multiplying by $1/\sigma_g$ from equation (14), or reflecting it from the line with slope $1/\sigma_g$ in the bottom half of Figure 14-8. Thus, once we locate an optimum return-risk point along the budget line in the top half of Figure 14-8, knowing σ_2 we can determine the corresponding portfolio mix of B and M in the bottom half of the diagram.

In order to locate the individual's equilibrium risk σ_T and expected rate of return $\bar{R}_T$ we must confront the technical budget constraint of Figure 14-8 with the individual's utility-function trade-off between risk and return. These preferences are represented by indifference curves such as those used in our analysis of consumption and investment. The shape of the curve depends on the nature of the investor's preferences between risk and return.

Tobin distinguished three kinds of preferences that an individual might have. These are shown in Figures 14-9 through 14-12. The first three figures represent *risk averters*. In these cases, the indifference curves have positive slopes, indicating that the person demands more expected return in order to be willing to take more risk. Figure 14-12 shows the indifference curves of a person who might be called a *risk lover*. The slope of her indifference curves is negative, showing that the risk lover is willing to take less return in order to be able to assume more risk.

The indifference curves shown in Figure 14-9 are representative of a subclass of risk averters known as *diversifiers*. As risk increases by equal increments, the diversifier demands increasing increments of return, so that her indifference curves are convex to her budget line. As usual in this kind of analysis, the diversifier will attempt to reach as high an indifference curve as

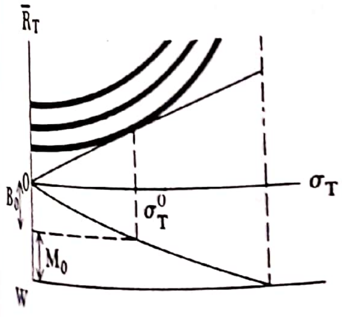

Figure 14-9 The "diversifier's" portfolio selection between risk and return.

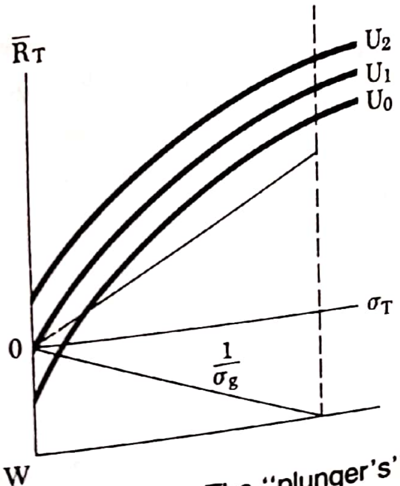

Figure 14-10 The "plunger's" portfolio selection: all money.

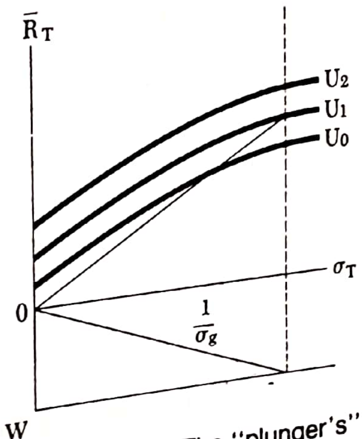

Figure 14-11 The "plunger's" portfolio selection: all bonds.

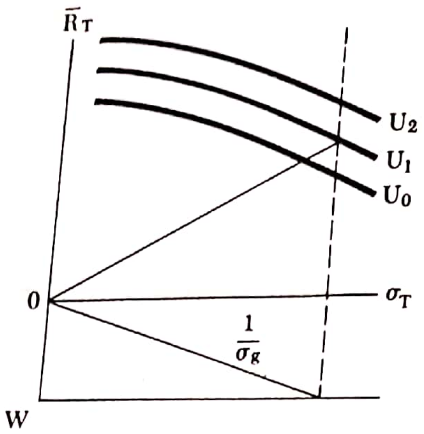

Figure 14-12 The "risk lover's" portfolio selection.

possible, given her budget constraint. Thus, the expected return and risk of her portfolio $\bar{R}_T$, σ_T will be determined by the point of tangency of the budget line with the highest possible indifference curve. Given the convex shape of her indifference curves, the diversifier is likely to reach an *interior equilibrium* at σ_T^0 holding both bonds B_0 and money M_0. Only part of this person's total wealth is put into bonds. This is why she is called a *diversifier*. Thus, in the case of the diversifier who demands increasing increments of return to induce her to take on constant increments of risk, the portfolio balance approach does away with the all-or-nothing version of the demand for money shown earlier in Figure 14-1.

Figures 14-10 and 14-11 show the indifference curves representative of a subclass of risk averters called *plungers*. The plunger will either not put her wealth into bonds at all or will put all of her wealth into bonds. In Figure 14-10, the plunger's indifference curves are steep relative to the budget line, so that she holds all money and no bonds. If her indifference curves are flat relative to the budget line as in Figure 14-11, she will hold all bonds, no money. This behavior would be consistent with the earlier regressive expectations model, but not with reality for most asset holders. Finally, Figure 14-12 shows the utility curve of the risk lover. She will attempt to maximize risk, and thus she, too, will put her entire wealth into bonds.

Since we observe empirically that the world is characterized by diversification, we can conclude that, in terms of the portfolio balance model, most asset holders are *diversifiers*. Thus, the situation shown in Figure 14-9, with indifference curves representing *increasing risk aversion,* is the basis for the portfolio balance model of the demand for money.

The Aggregate Demand for Money in the Portfolio Balance Model

We can now derive a demand function for money by varying the interest rate in Figure 14-9 and following the changes in the allocation of liquid wealth to

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bonds and money, particularly the latter. What happens in this model when interest rates rise? The result is shown in Figure 14-13. Since the slope of the budget line is $(r + \bar{g})/\sigma_g$, as r increases from r_0 to r_1 to r_2 the slope increases, and the line rotates upward. At any given level of risk, return will be increased as r rises. As r increases, the budget line touches successively higher indifference curves. This traces out the *optimum portfolio curve* connecting the points of tangency, shown in Figure 14-13. As r increases from a very low value, the diversifiers' tangency points move up and to the right, increasing both the expected rate of return and risk.

The progressively smaller increases in optimum risk from σ_T^0 to σ_T^1 to σ_T^2 in Figure 14-13 that come from continuing equal increases in r give successively smaller increases in the amount of wealth put into bonds. This is shown as B rises from B_0 to B_1 to B_2 in Figure 14-13. If, as r rises by constant increments, B rises by decreasing increments, then the demand for money must *decrease* by progressively smaller amounts as r increases, since $B + M$

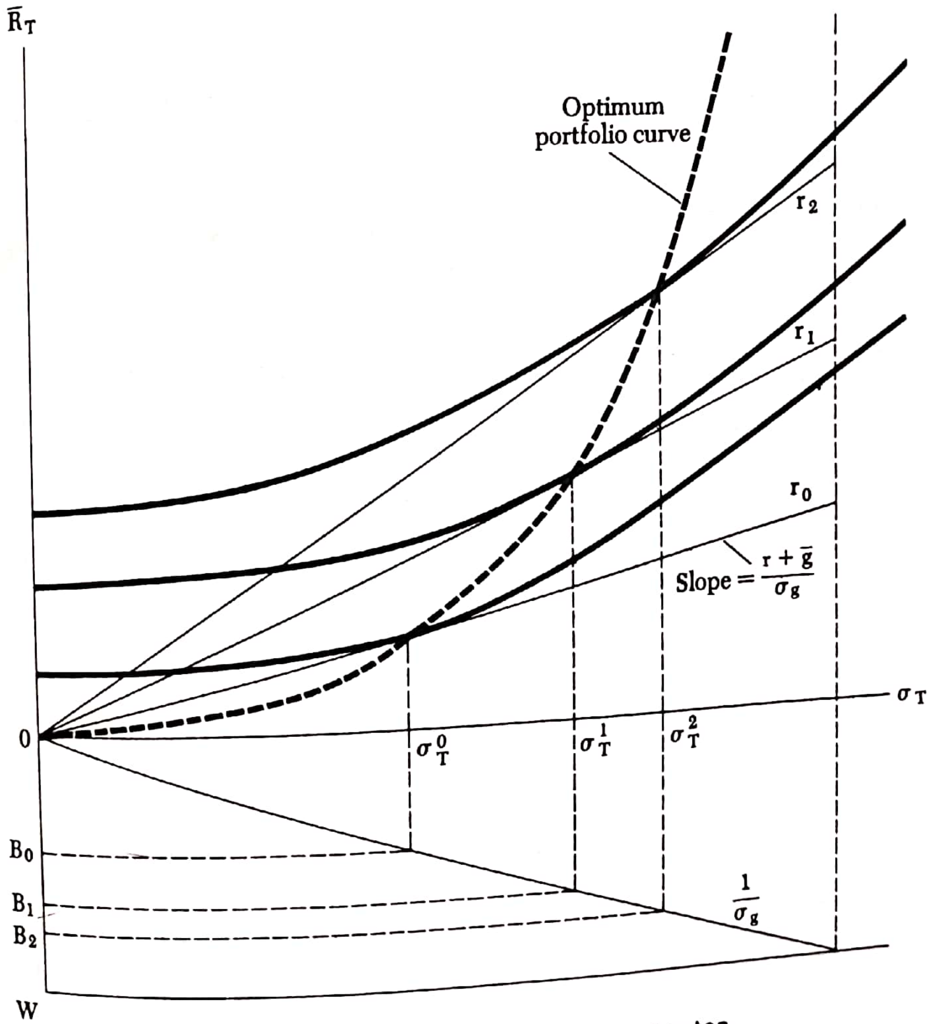

Figure 14-13 Portfolio selection with rising interest rates.

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equals a fixed W. This relationship between M and r is shown in Figure 14-14, which is simply the same demand-for-money curve that we derived in Chapter 4. Along this demand-for-money function, a given drop in r, measured by Δr in Figure 14-14, gives a bigger increase in the demand for money at a low interest rate—from point two to point three—than at a high interest rate—from point zero to point one.

The demand-for-money function of Figure 14-14 is drawn as $m(y_0)$, assuming a given level of real income. This is because the portfolio balance model is basically a theory of the speculative demand for money. It analyzes the allocation of a given amount of liquid wealth to bonds and money, depending on interest rates and expectations concerning the return and risk on capital gains. No reference is made in the model to a transactions demand for money. Thus, the portfolio balance model gives us a more satisfactory theory of the speculative demand for money than does the regressive expectations model, particularly in its explanation of diversification. In the next section we will review the inventory-of-money approach to transactions demand that has been developed by Baumol and Tobin. But first, let us look at the effects of changes in expected capital gains $\bar{g}$ and risk estimates, σ_g in the portfolio balance model.

An increase in expected capital gains $\bar{g}$ will have the same effect as an increase in the interest rate, rotating the budget line up and increasing the amount of liquid wealth held in bonds, decreasing the demand for money at any given interest rate. This would shift the demand curve in Figure 14-13 to the left; at any r, the demand for money is decreased.

What happens if estimates of risk change? The standard deviation σ_g of the probability distribution of Figure 14-6 may increase as a result of increasing uncertainty. This increase would rotate the budget line in the upper half of Figure 14-13 down (smaller slope) and also rotate the line in the lower half up by reducing $1/\sigma_g$. Since $\sigma_T = B \cdot \sigma_g$, an increase in σ_g means that the amount of bonds B yielding any given total risk falls. The increase in σ_g thus reduces bond holdings B in two ways. In the upper half of Figure 14-13, the downward rotation of the budget line reduces desired risk σ_T. Even with the original σ_g value, a reduction in σ_T would cause a drop in B. But the additional effect of the increase in σ_g, rotating upward the $1/\sigma_g$ line in the bottom half

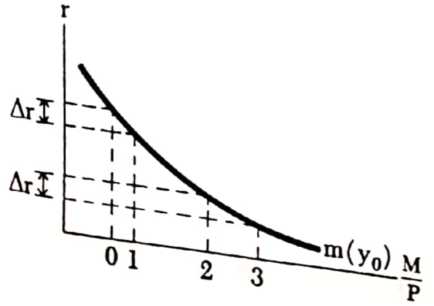

Figure 14-14 Demand for money.

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of Figure 14-13, would further increase the drop in B needed to reduce total risk σ_T while risk per bond σ_g is rising.

Tobin's model of portfolio balance provides a much firmer ground for the speculative demand for money by explaining why rational individual asset holders might hold their portfolios distributed among several assets of differing riskiness and expected return. It also explains why the speculative demand for money should be inversely related to the interest rate, in the way that was assumed in Chapter 4. We will now turn to the transactions demand to see that it, too, should be sensitive to interest rate changes.

TRANSACTIONS DEMAND FOR MONEY

In Chapter 4 we suggested that one principal motive for holding money is the need to smooth out the difference between income and expenditure streams. This *transactions motive* lies behind the transactions demand for money which is related to the level of income. The alternative to holding money, which is the means of payment and earns no return, is bonds, which earn a return but also incur transactions costs—brokerage fees—as one moves from money received as pay to bonds and back to money to make expenditures. In Chapter 4 we suggested that the higher the interest rate bonds earn, the tighter transactions balances should be squeezed to hold bonds, giving the transactions demand some degree of sensitivity to interest rate changes. Here we can develop this point a bit more thoroughly, using a model of the *interest-elastic* transactions demand originally developed separately by William Baumol and James Tobin.

Suppose an individual is paid monthly and spends a total amount of real income y on purchases spread evenly throughout the month. He or she has the option of holding transactions balances in money or in bonds. Bonds yield a given interest rate r if held for a month, and proportionately less than r if they are held for a shorter period. Exchanges of bonds for money incur *transactions costs,* which prevent the person from continuously exchanging bonds into money as purchases are made. So initially, the person will exchange most of the paycheck for bonds or, more realistically, deposit it in an interest-earning account. Then, periodically, he or she will exchange an amount of bonds into money or withdraw funds from the interest-earning account, and then run down those cash balances as purchases are made, until the time comes for the next exchange of bonds into money. The more transactions from bonds to money that the person makes, the longer will be the average bond holding, and therefore interest earned. But since transactions are costly, increasing transactions increases cost. So the number of transactions, or trips to the broker or savings bank, that the person makes will be determined by a trade-off between interest earnings on bonds and costs of transactions. The individual's

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average money holdings, or demand for real balances, will be determined by the number of transactions made, and the aggregate demand for money will reflect this representative individual's demand. We will now describe a basic model of this cost-minimization problem which the individual faces, and then derive the interest-elastic transactions demand for real balances from it.

The Optimum Number of Transactions

Our consumer anticipates spending y in real income over a month of length T (30 days, if you like) in a smooth flow of purchases. Let's assume that n transactions take place. The first will be conversion of $(n-1)/n$ percent of y into bonds, or deposit of $(n-1)/n$ percent into the interest-earning account, leaving $1/n$ percent of y in money to be spent in the first part of the month. The rest of the transactions, $n-1$ of them, will each convert y/n into cash. This breaks the month into n intervals each T/n long, with the person beginning each interval with y/n in cash and ending it with zero cash, rushing off to the bank or broker.

The cash flow of our representative consumer i is shown in the sawtooth pattern of Figure 14-15. There we show time on the horizontal axis and money holdings on the vertical. For concreteness, we assume five bonds-to-money transactions after the initial conversion of 5/6 of the paycheck into bonds. This divides the period T into six subperiods. The consumer begins each subperiod with y/n in real balances and ends each with zero. The average money holdings in this sawtooth pattern is obviously $\bar{m}_i = y/2n$. The corresponding pattern of bond holdings is shown in Figure 14-16. Initially, the consumer holds $[(n-1)/n] \cdot y$ in bonds and exchanges y/n for money each subperiod, ending with zero. In the example here, $(n-1)/n$ is 5/6.

The consumer's problem now is to determine the optimal number of transactions. To solve for optimal n, we can write the cost of money-holding—both transactions cost and interest foregone—as a function of the number of

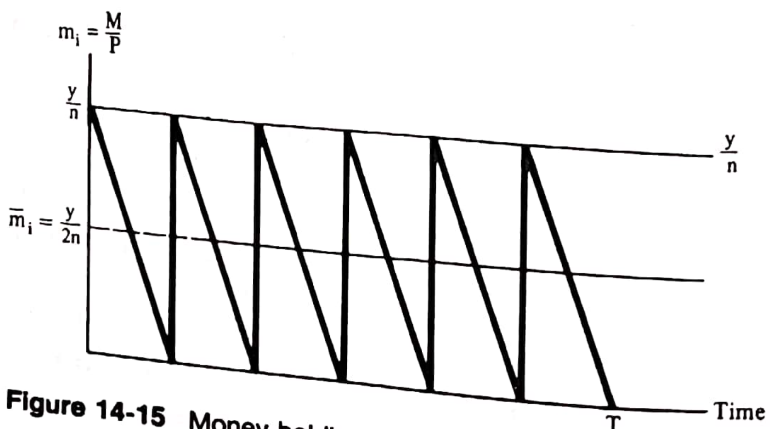

Figure 14-15 Money-holdings in the transactions model.

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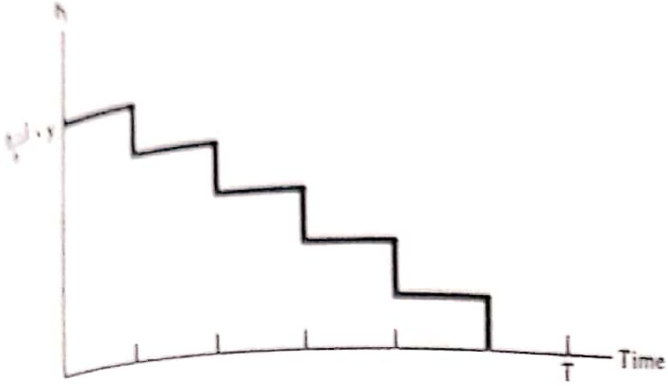

Figure 14-16 Bond holdings in the transactions model.

transactions n, and then find the cost-minimizing value of n. For simplicity, we assume that each transaction has a fixed cost a, so the transactions cost is na.

Calculation of the opportunity cost of foregone interest is a little more complicated. We assume that interest r is paid on bond or savings-account holdings proportional to the length of time they are held. Thus, the lost interest on money-holdings can be computed as follows. Initially, the consumer converts $[(n-1)/n]-y$ into bonds, keeping y/n in money. The interest lost on this money-holding is r for the full period T, or rTy/n. After one subperiod, another y/n is converted into money. That conversion out of bonds loses interest for $(n-1)/n$ percent of T, adding a loss of $rTy(n-1)/n$. The next conversion loses $rTy(n-2)/n$, and so forth. This gives us total interest lost as

$$\begin{aligned}\text{Interest cost} &= \frac{rTy}{n} + \frac{rTy(n-1)}{n} + \frac{rTy(n-2)}{n} + \cdots \\ &= \frac{rTy}{n}\left[1 + \frac{n-1}{n} + \frac{n-2}{n} + \cdots + \frac{1}{n}\right]. \qquad (17)\end{aligned}$$

The last term in the expansion in brackets is $[n-(n-1)]/n = 1/n$.

The problem now is to find the convergent value for the expansion in brackets in equation (17). To do this, factor $1/n$ out of the brackets to obtain

$$\text{Interest cost} = \frac{rTy}{n^2}[n + (n-1) + (n-2) + \cdots + 1].$$

The expression in brackets is now the sum of all numbers from 1 to n. This is given by $n(n+1)/2$. So the final expression for the interest cost for n transactions, including the initial one *into* bonds, is

$$\text{Interest cost} = \frac{rTy}{n^2}\cdot\frac{n(n+1)}{2} = \frac{rTy}{2}\left(1 + \frac{1}{n}\right). \qquad (18)$$

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We can now sum interest opportunity cost and transactions cost to obtain total cost TC:

$$TC = na + \frac{rTy}{2}\left(1 + \frac{1}{n}\right). \quad (19)$$

To solve for the cost-minimizing value of n, we set the partial derivative of total cost equal to zero:

$$\frac{\partial TC}{\partial n} = a - \frac{rTy}{2n^2} = 0.$$

Solving for optimal n, we obtain

$$n = \left(\frac{rTy}{2a}\right)^{1/2}. \quad (20)$$

The optimal number of transactions is the square root of $rTy/2a$. It increases with an increase in r, T, or y, and decreases with an increase in transactions cost a.

Individual and Aggregate Money Demand

Now that we have the solution for the number of transactions n, we can solve for the individual consumer's average money demand $\bar{m}_i$. From Figure 14-15, we have $\bar{m}_i = y/2n$. Using the optimal value of n from equation (20), we get for individual i's money demand

$$\bar{m}_i = \frac{y}{2}\left(\frac{2a}{rTy}\right)^{1/2} = \left(\frac{ay}{2rT}\right)^{1/2}. \quad (21)$$

The consumer's demand for real balances is the square root of $ay/2rT$. This is the well-known *square-root rule* of Baumol and Tobin. Transactions demand increases with the square root of y and decreases with the square root of r.

To move to aggregate money demand, we observe that for each representative consumer i whose money demand is given by equation (21), there must be someone on the other side of the money market. Suppose, for example, that consumer i buys goods from a representative firm, which periodically converts its money-holdings into bonds. This firm's pattern of bond and money-holdings would then mirror that of the consumer in Figures 14-15 and 14-16. In particular, we see that the firm's money-holdings would follow a sawtooth pattern exactly complementary to the consumer's pattern in Figure 14-15. This means that the firm's average money-holding would also be given by the square-root rule of equation (21).

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Aggregate money demand in the transactions model is the sum of the individuals and that of the firms on the other side of the market. This means we must double $\bar{m}_i$ in equation (21) to get the aggregate demand for real balances m:

$$\frac{M}{P} = m = 2\left(\frac{ay}{2rT}\right)^{1/2} = \left(\frac{2ay}{rT}\right)^{1/2}. \tag{22}$$

Aggregate demand follows the Baumol-Tobin square-root rule. The elasticity of demand for real balances with respect to income is 0.5; with respect to the interest rate it is −0.5. These numbers are not inconsistent with the empirical evidence summarized later in this chapter. Transactions costs enter through the parameter a in equation (22).

To summarize, an increase in the interest rate should reduce the transactions demand for real balances, for any given level of the income stream, by an elasticity of −0.5. This takes us back to the demand-for-money function of Chapter 4,

$$\frac{M}{P} = m(r, y); \qquad \frac{\partial m}{\partial r} < 0, \qquad \frac{\partial m}{\partial y} > 0. \tag{23}$$

The transactions demand for money should respond to a change in the interest rate through a change in the number of bonds-to-money conversions. Thus, money demand is interest-sensitive even if all demand is for transactions. Any speculative component may add to this sensitivity, but speculative demand is not necessary for r to enter the money demand equation. One might even say that the division of money-holdings into speculative and transactions balances is largely a matter of analytic convenience.

Precautionary Demand in a Stochastic Setting

The transactions demand for money will exist if incomes and expenditures are not perfectly coordinated, whether or not they are deterministic. The *precautionary demand* is a fairly straightforward extension to a stochastic setting where the surplus or deficit of current income over expenditure is random. At each moment there may be a discrepancy between income and spending. Perhaps a worker's income rises because she has a chance to work overtime or falls because she is temporarily laid off. On the expenditure side, a shopper may come across a good "limited time only" offer, or her desired purchase may be out of stock.

On average, expenditure must be no more than income. The dilemma for the individual is that, on the one hand, it is costly if he or she does not have enough resources readily available to make a sudden purchase. Perhaps obtaining liquid funds by trading in bonds suffers a transaction cost, or the

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individual suffers a loss of utility when he or she cannot afford to pay for something on the spot (e.g., the acute embarrassment brought on when your date orders the most expensive item on the menu). This problem is alleviated by holding a stock of money as a sort of homemade insurance. One would hold more liquid funds if the cost of running short increased, or if the probability of doing so went up. Were the variability of the net surplus to increase with the expenditure, then money-holdings would also rise with income.

On the other hand, money pays little or no interest, so liquidity is achieved at an opportunity cost proportional to the size of the money stock and the interest rate. The money demand equation is the outcome of the trade-off between the expected losses from illiquidity and foregone interest, as in the transactions model.

We will now turn to another major perspective on the demand for money—Friedman's view of money as a consumer's and producer's good. This will lead us back again to the original demand-for-money equation (23).

MONEY AS A CONSUMER'S AND PRODUCER'S GOOD

The money demand models discussed so far, due mainly to Keynes and Tobin, draw an important distinction between transactions and speculative demands for money. Friedman, however, develops the demand for money within the context of the traditional microeconomic theories of consumer behavior and of the producer's demand for inputs. Consumers hold money because it yields a utility—the convenience of holding the means of payment rather than making frequent trips to the broker and risking losses on bonds. Their demand for money should be a demand for real balances, just as any consumer demand should be a demand for real consumer's goods, as opposed to their money value, in the absence of money illusion. This demand for real balances should depend on the level of real income. It should also depend on the returns to other ways of holding assets such as bonds or consumer durables, much as the demand for one kind of fruit should depend on the prices of other kinds.

Producers hold money as a productive asset that smooths payments and expenditure streams. Just as their demand for real capital services depends on the level of real output and the relative price of capital, as shown in Chapter 13, their demand for real balances should depend on real output (or income), and the relative returns on other ways of holding wealth.

The approach gives us a demand function for real balances,

$$\frac{M}{P} = m = m(y, r_1, \ldots, r_j, \ldots, r_J), \tag{24}$$

where $r_1, \ldots, r_J$ are the rates of return on all assets that are alternatives to money. If the ratio between the demand for real balances and real income is relatively trendless through time, and depends at any given point in time on the returns to alternative assets, we have Friedman's quantity theory version of (24):

$$\frac{M}{P} = k(r_1, \ldots, r_J) \cdot y, \tag{25a}$$

or

$$\frac{m}{y} = k(r_1, \ldots, r_J). \tag{25b}$$

In fact, as we will see shortly, the elasticity of m with respect to changes in y may well be about unity, so that the ratio between m and y is roughly constant along trend, and (25) is a good approximation to (24).

To fill out the demand function, we can include the rate of return on bonds and on durable goods as examples of the more complete list of alternative assets that might be relevant substitutes for money.

As we have already shown at some length in this chapter, as the rate of return on bonds rises, the demand for money falls. Rather than distinguish between transactions demand and speculative demand, we can simply note that as the expected total return on bonds rises, the demand for bonds should rise and the demand for money should fall. Earlier, we developed an expression for the expected return on bonds, equation (5):

$$e = r + \frac{r}{r^e} - 1.$$

Since bonds are a relevant substitute for money, we would insert r and r^e into the demand function (25). If $r^e = f(r)$, we can condense this expression of the dependency on the bond rate by simply including r in (25).

The Effect of the Rate of Inflation

Durable goods—producers' or consumers'—also serve as alternative assets to money. In Chapter 13, we saw that the real interest rate $(r - \dot{P})$ enters the demand function for purchases of producers' durables (investment demand). Here we explore the relation between consumer durables and money demand through the rate of inflation. As the price level rises, the value, or purchasing power, of a stock of durable goods remains roughly constant as durable goods prices rise along with the general price index. On the other hand, the purchasing power of money falls with an increase in prices, so that an increase in the *expected rate of inflation* should cause a shift out of money and bonds and

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Since $\partial k/\partial r$ and $\partial k/\partial \dot{P}$ are both negative, $\partial v/\partial r$ and $\partial v/\partial \dot{P}$ are both positive. An increase in either the interest rate or the rate of inflation should cause people to economize on money-holdings since these are the rates of return on the alternative assets, bonds, and durable goods. This would result in an increase in velocity as money demand falls relative to GNP.

In the long-run U.S. data, the velocity of money seemed, on balance, to decline along trend up to World War II. Since then it has risen, along with interest rates, fairly steadily. This suggests that, over the long run, the ratio of y to m is rather stable, as is the ratio of consumption to income, c/y. The question of the relative stability of the y/m ratio v and the $c - y$ relationship is, as we will see in Chapter 16, one of the points at the heart of the *Keynesian-Monetarist* controversy.

If the short-run elasticity of v with respect to changes in interest rates were very low, we could approximate $v(r, \dot{P})$ in (28) by a constant $\bar{v}$, giving a direct relationship between movements in m and y; $y = \bar{v}m$. However, there is substantial empirical evidence that the interest elasticity of the demand for money is not insignificant in the short run. This suggests, through equation (22), that the interest elasticity of velocity is not insignificant so that, in fact, both the product and money market equations are needed to predict movements in nominal and real GNP. We will return to this point in some detail in the discussion of monetary and fiscal policy in Chapter 16.

EMPIRICAL ESTIMATES OF INCOME AND INTEREST ELASTICITIES

In the absence of substantial shifts in expected rates of inflation, which are not fully compatible with a static model of income determination which deals with movements from one equilibrium price level to another, the demand-for-money function that emerges from the analysis of this chapter is the familiar

$$\frac{M}{P} = m = m(r, y) \tag{29}$$

of Chapter 4.

The y term in the demand function represents transactions demand or, in Friedman's terms, the increasing demand for money as a producer's and consumer's good, through an income effect, as income rises. The r term represents the interest elasticity of both the transactions demand for money and the speculative demand through Tobin's portfolio balance model. It also represents a potential substitutability against bonds in production and consumption decisions.

There have been many investigations into the values of the elasticities of the demand for *real money*—currency plus demand deposits, $M1$ or $M1$ plus time deposits, $M2$, deflated by the price level P as in equation (29)—with

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respect to interest rate and income changes. There are many continuing controversies concerning the values of these elasticities and the proper form of the demand-for-money function. However, representative elasticities of the demand for real balances from Goldfeld's 1973 article are about 0.7 with respect to changes in real income, and −0.25 with respect to changes in r. Here r is a short-term interest rate—a three-month commercial paper rate, a three-month treasury bill rate, or a time deposit rate. With the money stock $M1$ at about $750 billion in 1987, GNP at about $4.4 trillion, and the short-term interest rate at about 6 percent, these elasticities imply that a $65 billion GNP increase—1.5 percent—would raise the demand for money by about $8 billion, and a 1-point drop in short-term interest rates—17 percent—would increase the demand for money by about $32 billion.

Most demand-for-money estimates suggest that demand changes lag slightly behind interest rate changes. Thus, if interest rates fell from 5 to 4 percent, causing an increase of $30 to $40 billion in the desired holdings of money, the estimates suggest that about 30 percent of the discrepancy between actual and desired money-holdings would be eliminated in one quarter, 50 percent in two quarters, and 75 percent in a year. This means that if an increase in y or decrease in r increases the level of desired equilibrium real balances from $(M/P)_0$ to $(M/P)^*$ at time t_0 in Figure 14-17, the path of actual holdings of $M/P = m$ will tend to follow the dashed adjustment path toward the final desired level of real balances, $(M/P)^*$.

This partial adjustment pattern leads to the same kind of overreaction of interest rates to changes in the money supply that we see in traditional microeconomic analysis, in which demand changes yield larger short-run than long-run changes in prices, and vice versa for output. In Figure 14-18 the demand-for-money curve $M(y_0)$ is the long-run curve reflecting the long-run interest elasticity mentioned above as about −0.25. The intersection of $M(y_0)$ with the given initial money supply, $(M/P)_0$, at point E_0, gives an initial equilibrium interest rate r_0. Through E_0 we can also draw a short-run demand function $m(y_0)$ that reflects the partial short-run adjustment mechanism of Figure 14-17. This shows the one-period reaction of demand to interest rate

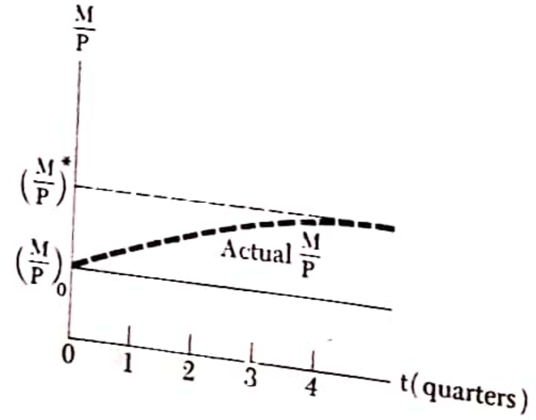

Figure 14-17 The adjustment path of actual holdings of M/P.

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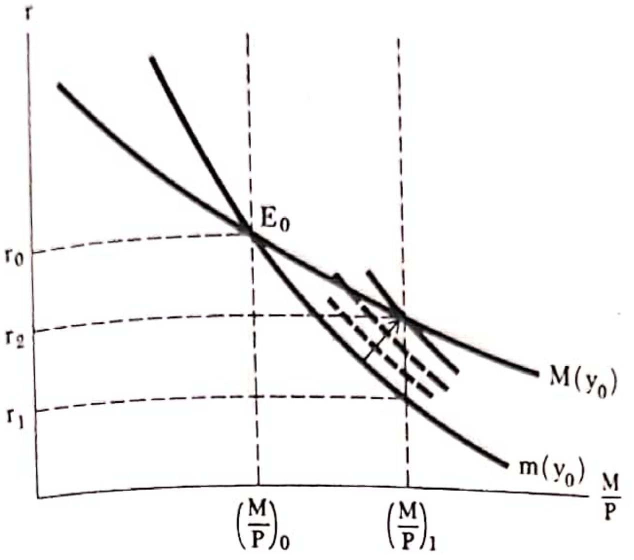

Figure 14-18 Long-run and short-run demand for money and effects of an increase in the money supply.

changes, while the $M(y_0)$ curve shows the long-run relationship between M/P and r.

Now if the money supply is increased to $(M/P)_1$ the interest rate will initially fall to r_1 along the short-run $m(y_0)$ function, holding y constant throughout this partial analysis, that is, assuming a vertical *IS* curve. As time passes, and the actual demand shifts up toward equilibrium along the dashed path of Figure 14-17, the interest rate will rise toward r_2, with the short-run demand function shifting up, shown as the dashed short-run functions of Figure 14-18. Eventually, holding y constant at y_0, the interest rate will settle at r_2, rising from the short-run level of r_1 but below the initial equilibrium r_0.

If the level of income has also risen during the process as the *LM* curve shifts along an actual nonvertical *IS* curve, the interest rate will end up higher than r_2, on a higher $M(y_1)$ function, but lower than the initial r_0. Thus, since the demand for money is less elastic in the short run than in the long run, owing to the partial adjustment process of Figure 14-17, we can expect an increase in the money supply to push interest rates down somewhat more in the short run, one quarter, than in the slightly longer run, one year.

Since this chapter's review of the several approaches to the demand for money has not substantially altered the demand function we began with in Chapter 4, there is no need to return to the static model of income determination here. This chapter has illustrated another kind of channel that economic research follows. In this chapter the emphasis has been on developing theoretical explanations—or, put more simply, ways to understand common empirical observations such as people holding positive amounts of two assets with different rates of return at the same time (diversification), and the interest

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elasticity of transactions balances. After we briefly discuss in Chapter 15 the supply of money—a quantity taken as given up to now—we will return to some of the important points in the analysis of the demand for money in Chapter 16.

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