ASUTOSH COLLEGE
Department of Mathematics
Assignment (4th Semester) (Hons)
POWER SERIES

Choose the correct option with proper justification.

1. If R be the radious of convergence of the Power series $\sum a_n x^n$ then that of the series $\sum (n+1) a_n x^n$ is
   a) R^2 b) less than R c) R d) grater than R.

2. The radious of convergence of the Power series $\sum \frac{n^n}{n!} x^n$ is
   a) e b) $\frac{1}{e}$ c) e^2 d) does not exist.

3. The series $\sum \frac{x^n}{n}$, where x is a real variable. Then
   a) converges if $|x|<1$ and diverges if $|x| \geq 1$
   b) converges if $|x| \leq 1$ and diverges if $|x|>1$
   c) converges if $|x| \leq 1$, $x \neq 1$ and diverges otherwise.
   d) none of the above.

4. consider a power series $\sum_{n=0}^{\infty} a_n x^n$, where $a_0 = 1$, $2 \leq a_n \leq 3$ for $n \geq 1$. Then
   a) radious of Convergence of the Series is 2.
   b) radious of Convergence of the Series is $\frac{1}{3}$
   c) radious of Convergence of the Series is 1
   d) none of the above.

5. The Power series $\sum_{n=1}^{\infty} \frac{[2+(-1)^n]^n}{3^n} x^n$ Converges
   a) only for $x=0$.
   b) for all $x \in \mathbb{R}$
   c) only for $-1<x<1$
   d) only for $-1<x \leq 1$.

6. Consider the Power series $\sum_{n=1}^{\infty} a_n x^n$ where a_n = number of divisors of n^{50}. Then the radius of convergence of $\sum_{n=1}^{\infty} a_n x^n$ is

a) 1 b) 50 c) $\frac{1}{50}$ d) 0

7. The Power Series $\sum_{n=0}^{\infty} 2^{-n} x^{2n}$ converges if

a) $|x| \leq 2$ b) $|x| < 2$ c) $|x| \leq \sqrt{2}$ d) $|x| < \sqrt{2}$

8. The Power Series $\sum_{n=0}^{\infty} 3^{-n} (z-1)^{2n}$ converges if

a) $|z| \leq 3$ b) $|z| < \sqrt{3}$ c) $|z-1| < \sqrt{3}$ d) $|z-1| \leq \sqrt{3}$.

9. If the Power series $\sum_{n=1}^{\infty} a_n x^n$ converges for $x = 3$, then the series $\sum_{n=1}^{\infty} a_n x^n$

a) converges absolutely for $x = -2$
b) converges but not absolutely for $x = -1$
c) converges but not absolutely for $x = 1$
d) diverges for $x = -2$.

10. The radius of converges of the Power series $\sum_{n=0}^{\infty} 2^{2n} x^{n^2}$ is

a) $\frac{1}{4}$ b) 1 c) 2 d) 4.

11. Let $a_n = \begin{cases} \frac{1}{3^n}, & n \text{ is prime} \\ \frac{1}{4^n}, & n \text{ is not Prime} \end{cases}$. Then the radius of convergence of the Power series $\sum_{n=1}^{\infty} a_n x^n$ is

a) 4 b) 3 c) $\frac{1}{3}$ d) $\frac{1}{4}$

12. The set of all x at which the Power series $\sum_{n=1}^{\infty} \frac{n}{(2n+1)^2} (x-2)^{3n}$ converges is

a) $[-1, 1]$ b) $[-1, 1)$ c) $[1, 3)$ d) $[1, 3]$

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13. The radius of convergence of the Power Series $\sum_{n=0}^{\infty} a_n 2^n$, where $a_0 = 1$, $a_n = 3^n a_{n-1}$, $n \geq 1$, is

a) 0 b) $\sqrt{3}$ c) 3 d) ∞.

14. If R_1, R_2 be the radii of convergence of the Power Series $\sum_{n=0}^{\infty} a_n x^n$ and $\sum_{n=0}^{\infty} b_n x^n$ respectively, then the radius of convergence of the Series $\sum_{n=0}^{\infty} (a_n + b_n) x^n$ is

a) $R_1 + R_2$ b) $R_1 R_2$ c) $\max(R_1, R_2)$ d) $\min(R_1, R_2)$.

15. Let $R > 0$ be the radius of convergence of the Power Series $a_0 + a_1 x + \cdots$. If R' be the radius of Convergence of the Power Series $a_0 x + \frac{a_1}{2} x^2 + \frac{a_2}{3} x^3 + \cdots$ Then

a) $R' = \frac{R}{2}$ b) $R' = 2R$ c) $R' = R$ d) $R' = R^2$.

16. The radius of Converges of the Power series $\sum_{n=1}^{\infty} n^{-\sqrt{n}} x^n$ is

a) 1 b) $\frac{1}{2}$ c) 2 d) ∞.

17. If r and R be the radii of Convergence of the two Power series $\sum n x^n$ and $\sum x^n$ resp. Then which of the following is true?

a) $0 < r < R = 1$ b) $r = R = 1$ c) $r > R > 1$ d) $0 < r < 1 < R$.

18. Given That the radius of Convergence of the Power Series $\sum a_n x^n$ is 2. Then That of the Series $\sum a_n x^n$ is

a) 2 b) $\frac{1}{2}$ c) 1 d) $2^{1/n}$.

19. The series $\sum (-1)^n a_n$ be convergent. Then the series $\sum a_n x^n$

a) is absolutely convt when $|x|<1$

b) is convt. but not absolutely convt. in $|x|<1$

c) is not convt. but absolutely convt in $|x|<1$

d) neither converges nor ~~diverges~~ absolutely converges in $|x|<1$.

20. Find out which of the following series converge uniformly for $x \in (-\pi, \pi)$

a) $\sum_{n=1}^{\infty} \frac{e^{-n|x|}}{n^3}$

b) $\sum_{n=1}^{\infty} \frac{\sin nx}{n^5}$

c) $\sum_{n=1}^{\infty} \frac{x^n}{n^n}$

d) $\sum_{n=1}^{\infty} \frac{1}{((x+\pi)n)^2}$.