CHAPTER 9

INTERFERENCE OF LIGHT

9.1 INTRODUCTION :

When two or more disturbances arrive at a point in space simultaneously the resultant disturbance at that point, according to the *principle of superposition,* is given by the *vector* sum of the disturbances which are assumed to be small. The differential wave Eq. (8.3-1) governing the propagation of a disturbance in the form a wave is a second order, linear homogeneous equation. If ψ_1 and ψ_2 are two solutions then $\psi_1 + \psi_2$ is also a solution to this equation. It is in accordance with the principle of superposition. An important consequence of the principle of superposition is that when two beams of light intersect, the propagation of each beam is completely unaffected by the presence of the other. However, in the region of crossing both the beams act simultaneously and we expect a change in intensity. A case of utmost importance occurs when monochromatic waves of light from two sources proceed almost in the same direction and superpose at a point either in same or in opposite phase. Then the intensity of light at that point will be maximum or minimum according as the waves meet the point in the same or in opposite phase. This phenomenon is known as *interference of light*. This phenomenon requires for its explanation that light must have a wave nature.

9.2 THEORY OF INTERFERENCE :

Suppose a narrow slit S is illuminated with monochromatic light of wavelength λ. There are two other slits S_1 and S_2 equidistant from S. According to Huygens' principle cylindrical wavelets spread out from the slit S. As $SS_1 = SS_2$, the wavelets will reach S_1 and S_2 at the same time instant. Hence new secondary wavelets will start from S_1 and S_2 and diverge towards the screen. These wavelets start with equal phase and hence S_1 and S_2 may be considered as coherent sources. Let us represent the complex disturbances constituting light waves at S_1 by

$$y = a_1 e^{i\omega t} \quad \text{...(9.2-1)}$$

In travelling from S_1 to any point P on the screen the phase of the wave changes by $\frac{2\pi}{\lambda} S_1P = \frac{2\pi}{\lambda} x_1 = kx_1$. Hence at the instant t the

Similarly, the disturbance at P due to light from S_2 may be represented by,

$$y_2 = a_2 e^{i(\omega t - kx_2)} \qquad \ldots(9.2\text{-}3)$$

$\therefore$ The resultant disturbance at P, according to the principle of superposition, is given by

$$y = y_1 + y_2 = e^{i\omega t}\left[a_1 e^{-ikx_1} + a_2 e^{-ikx_2}\right]$$

$$= e^{i\omega t}\left[a_1(\cos kx_1 - i\sin kx_1) + a_2(\cos kx_2 - i\sin kx_2)\right]$$

$$= e^{i\omega t}[a - ib] = e^{i\omega t}.\sqrt{a^2+b^2}.e^{-i\phi}\ ;\ \phi = \tan^{-1}\frac{b}{a} \qquad \ldots(9.2\text{-}4)$$

where $a = a_1 \cos kx_1 + a_2 \cos kx_2$

$b = a_1 \sin kx_1 + a_2 \sin kx_2$

Therefore,

$$y = Ae^{i(\omega t - \phi)}$$

where

$$A^2 = a^2 + b^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos k(x_2 - x_1) \qquad \ldots(9.2\text{-}5)$$

Here A is the amplitude of the resultant disturbance. Hence the intensity of light at P is given as

$$I \propto A^2$$

$$= a_1^2 + a_2^2 + 2a_1 a_2 \cos\delta \qquad \ldots(9.2\text{-}6)$$

where $\delta = k.(x_2 - x_1) = \frac{2\pi}{\lambda}(x_2 - x_1)$ is the phase difference between the interfering waves. Obviously this phase difference depends on the path difference $x_2 - x_1$ and hence on the position of the point P. Now if

$\delta = 2m\pi$ or, $x_2 - x_1 = 2m\frac{\lambda}{2}$; $m = 0,1,2,3\ldots$, then intensity I is maximum and proportional to $(a_1 + a_2)^2$. Thus when the path difference of the point P from the two sources is even multiple of $\lambda/2$ the intensity of light at that point becomes maximum and we get a bright band there. This is known as *constructive interference.* When $m = 0$ we get the central bright band at C for which the path difference is zero.

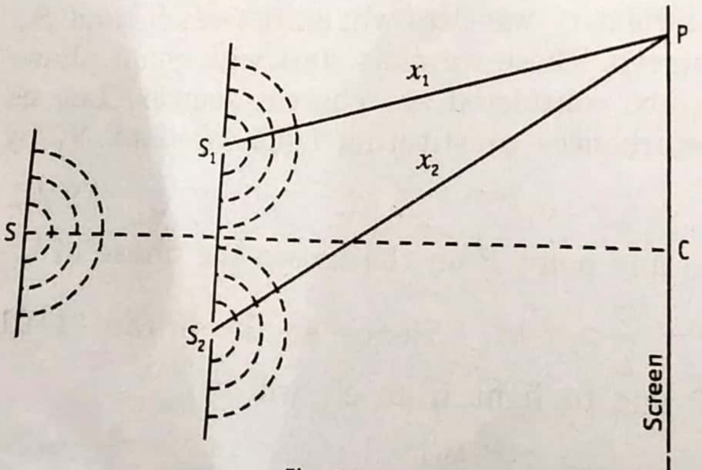


Fig. 9.2-1

On the other hand, if $\delta = (2m + 1)\pi$ or, $x_2 - x_1 = (2m + 1)\lambda/2$; $m = 0,1,2,3\ldots$ then intensity I is minimum and proportional to $(a_1 \sim a_2)^2$. Thus when the path difference of the point P from the two sources is odd multiple of $\lambda/2$ the intensity of light at that point is minimum and we get a nearly dark band there. This is known as *destructive interference.*

When $a_1 = a_2$, the amplitudes of the disturbances are equal, the minimum intensity is zero and we get alternately bright and completely dark bands as we go away from the central bright band. The phenomena which are produced on the screen at a particular instant of time when waves from the two coherent sources S_1 and S_2 superpose, are illustrated in the Fig. 9.2-2. C is a point on the screen which is equidistant from S_1 and S_2 and hence its path difference is zero. The waves from the

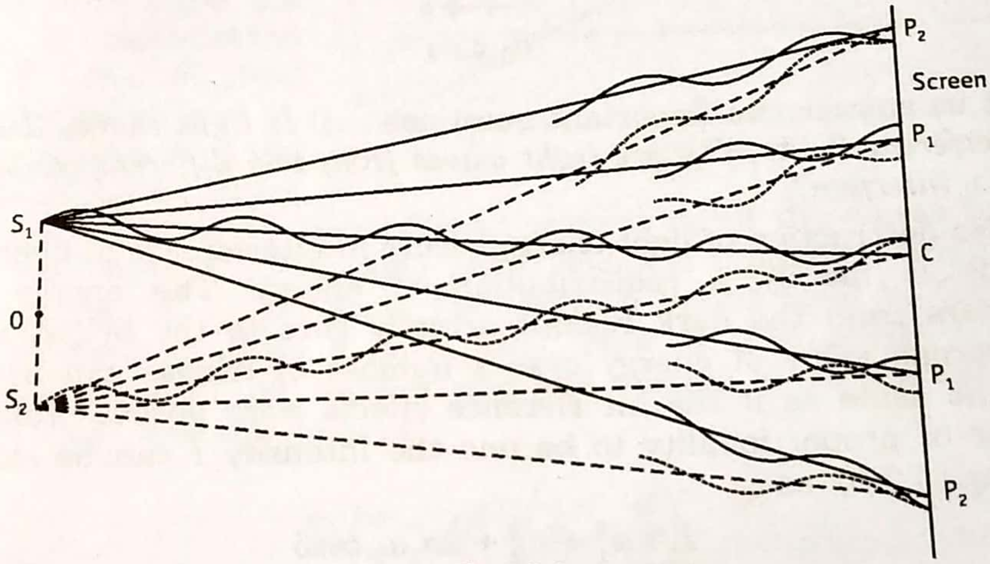


Fig. 9.2-2

sources S_1 and S_2 (whose phase difference is permanently zero) meet at C in the same phase (crests of both the waves fall at C). At this point the resultant amplitude is sum of the amplitudes of individual waves and hence we get a bright band there, known as the *central bright band.*

If we take a point P_1 on the screen whose path difference from S_1 and S_2 is $\lambda/2$, then the waves meet at P_1 in the opposite phases (the crest of the wave from S_1 falls on the trough of the wave from S_2) and we get a dark band there known as *first order dark band.*

Again the path difference of another point P_2 on the screen from S_1 and S_2 is $2\lambda/2$ and the waves meet there in the same phase (troughs of both the waves fall there) which is, therefore, a position of the next bright band known as the *first order bright band.* Thus as we proceed from C, the path difference changes from odd to even multiples of $\lambda/2$ causing respectively an alternation of minimum and maximum intensity of light on both sides of the central band. The intensity distribution curve with $a_1 = a_2 = a$ is shown in Fig. 9.2-3.

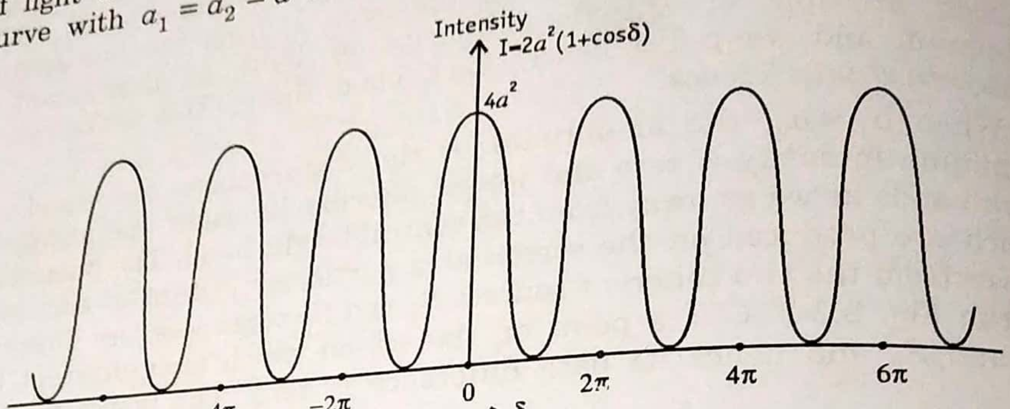


Fig. 9.2-3

Let us answer two important questions : (i) *Is light energy destroyed by interference?* (ii) *Why are light waves from two different candles not seen to interfere?*

(i) No destruction of light energy occurs in interference of light. What happens is merely a redistribution of energy. The energy which disappears from the dark regions actually goes to the bright regions. The average value of energy over a number of fringes can be shown to be the same as if the interference effects were absent. Taking the constant of proportionality to be one the intensity I can be expressed from Eq. (9.2-6) as

$$I = a_1^2 + a_2^2 + 2a_1a_2\cos\delta$$

where a_1 and a_2 are the amplitudes of the waves and δ is the phase difference.

$$\therefore\ I_{average} = \frac{\int_0^{2\pi} I\,d\delta}{\int_0^{2\pi} d\delta} = a_1^2 + a_2^2$$

= sum of intensities of individual waves.

Thus we can say that there is no destruction of energy or the principle of conservation of energy is not violated in the phenomenon of interference.

(ii) When light waves from two separate candles meet at a point, the point will be bright or dark according as the waves meet in the same or opposite phases. This phase difference depends on the initial phase difference between the sources and the optical path difference of the point from the two sources. The path differenc...

in question remains constant but as two different candles cannot be coherent, the initial phase difference goes on changing very rapidly with time. As a result at every point there is rapid alternation of brightness and darkness and hence we get general illumination.

9.3 WIDTH AND SHAPE OF INTERFERENCE FRINGES :

Fringe width :

Let us consider two coherent sources S_1 and S_2 which are sending monochromatic light of wavelength λ (Fig. 9.3-1). The point C on the screen is equidistant from S_1 and S_2 and hence the waves from S_1 and S_2 arrive at C at the same time. If the phase difference between the two coherent sources is nil the waves from S_1 and S_2 will meet at C in same phase and will produce a bright band known as central bright band. Let the mth bright band be formed at P whose distance from C is x_m(say). Now in Fig. 9.3-1, D is the separation beween the sources and the screen; d is the separation between the coherent sources.

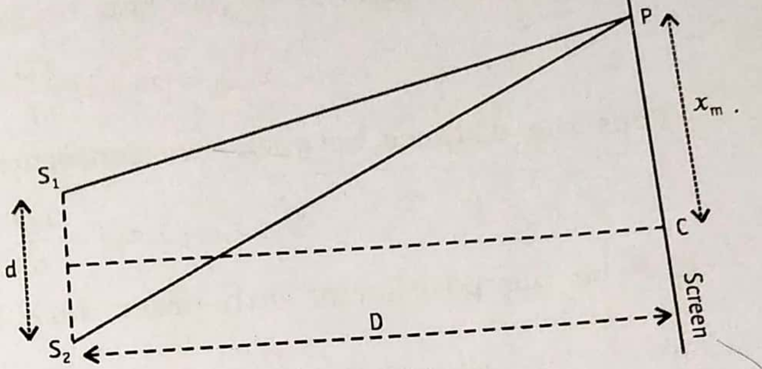


Fig. 9.3-1

$$\therefore \quad S_2P^2 = D^2 + \left(x_m + \frac{d}{2}\right)^2$$

or,

$$S_2P = D\left[1+\left(\frac{x_m + \frac{d}{2}}{D}\right)^2\right]^{\frac{1}{2}}$$

Since usually $D >> d$,

$$S_2P \approx D\left[1+\frac{1}{2}\left(\frac{x_m + \frac{d}{2}}{D}\right)^2\right] \quad \ldots(9.3\text{-}1)$$

Similarly,

$$S_1P \approx D\left[1+\frac{1}{2}\left(\frac{x_m - \frac{d}{2}}{D}\right)^2\right] \quad \ldots(9.3\text{-}2)$$

Now for mth order bright fringe at P,

$$S_2P - S_1P = m\lambda$$

Using Eqs. (9.3-1) and (9.3-2) we get,

$$\frac{x_m . d}{D} = m\lambda; \text{ or, } x_m = m\frac{\lambda D}{d}$$

Similarly, the distance of $(m+1)$th bright fringe from C will be

$$x_{m+1} = (m+1)\cdot\frac{\lambda D}{d}$$

Thus the distance between two consecutive bright bands would be

$$\beta = x_{m+1} - x_m = \frac{\lambda D}{d} \qquad ...(9.3\text{-}3)$$

If P be the position of mth order dark band then

$$x_m = (2m+1)\frac{\lambda}{2}\cdot\frac{D}{d} \text{ and } x_{m+1} = (2m+2+1)\frac{\lambda}{2}\cdot\frac{D}{d}$$

Hence

$$\beta = x_{m+1} - x_m = \frac{\lambda D}{d} \qquad ...(9.3\text{-}4)$$

Eq. (9.3-3) and (9.3-4) show that the distances (β) between two consecutive bright or dark bands are equal. β is known as *fringe width*. Measuring β, d and D the expression for β can be used to measure λ.

Shape of interference fringes :

An idea regarding the shape of interference fringes can be obtained by finding the locus of points having a constant path difference from the sources (slits) S_1 and S_2. Refering to Fig. 9.3-2 we choose the mid point O between the slits as the origin of a coordinate system. Let the x-axis be along OX and y-axis be perpendicular to the plane containing the slits. For any point $P(y, x)$ we can write

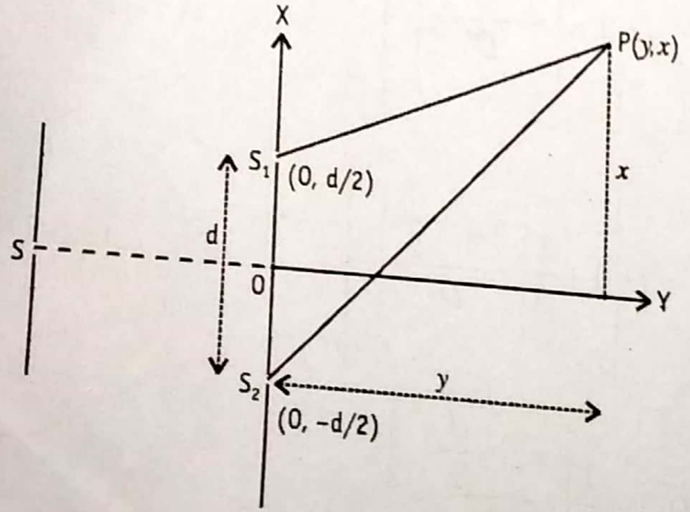


Fig. 9.3-2

$$S_1P^2 = y^2 + \left(x - \frac{d}{2}\right)^2 \text{ and}$$

$$S_2P^2 = y^2 + \left(x + \frac{d}{2}\right)^2$$

where d is the separation between the slits. The path difference,

$$\Delta = S_2P - S_1P$$

or,

$$\Delta + \left[y^2 + \left(x - \frac{d}{2}\right)^2\right]^{\frac{1}{2}} = \left[y^2 + \left(x + \frac{d}{2}\right)^2\right]^{\frac{1}{2}}$$

Squaring both sides and rearranging we get,

$$2\Delta\left[y^2 + \left(x - \frac{d}{2}\right)^2\right]^{\frac{1}{2}} = 2xd - \Delta^2$$

Squaring again and rearranging we get

$$\frac{x^2}{\Delta^2/4} - \frac{y^2}{\left(d^2 - \Delta^2\right)/4} = 1 \qquad \ldots(9.3\text{-}5)$$

Thus the loci of points of constant path difference Δ in xy plane are hyperbolae with S_1 and S_2 as foci on x-axis. The eccentricities of the hyperbolae are given by

$$e = \left[\frac{\Delta^2}{4} + \frac{d^2 - \Delta^2}{4}\right]^{\frac{1}{2}} \div \frac{\Delta}{2} = \frac{d}{\Delta}$$

In optical experiments the path difference $\Delta \sim 10^{-8}$ cm and $d \sim 10^{-2}$ cm. Therefore, e is very high and as a consequence the hyperbolae become practically straight lines given by

$$y = \pm\left[\frac{d^2 - \Delta^2}{\Delta^2}\right]^{\frac{1}{2}} . x$$

If instead of slits we use two coherent point sources S_1 and S_2 then in three dimensional space the loci of maxima i.e., bright fringes of different order numbers will form a system of confocal hyperboloids with S_1 and S_2 as foci (Fig. 9.3-3). If a screen is placed parallel to the line joining S_1 and S_2 then short straight line fringes (parallel to the length of the slits placed at S_1 and S_2) will be obtained. If the screen is placed perpendicular to the line joining S_1 and S_2, we shall get a number of alternately bright and dark concentric circles with their common

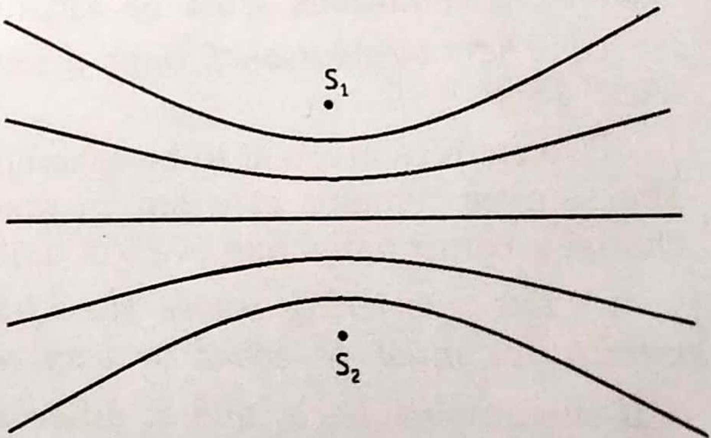


Fig. 9.3-3

centre lying on the point of intersection of the line S_1S_2 with the screen. These fringes are called **non-localized fringes** for they can be obtained on a screen wherever it is placed.

9.4 INTERFERENCE WITH WHITE LIGHT AND COLOUR EFFECT :

The distance of the *m*th bright fringe from the central one is given by

$$x_m = m \cdot \frac{\lambda D}{d}$$

where d is the separation between the two coherent sources and D is the distance of the screen from the sources. x_m is a function of wavelength λ. So the fringes of different colours will be in step only for the central band ($m = 0$). In this case $x_m = 0$ and bright fringe of all wavelengths will coincide and the central band will be white with white light. For higher order bright fringes ($m > 0$) x_m will be greater for a light of longer wavelength and less for a light of shorter wavelength. As the wavelength λ_r for red light is longer than the wavelength λ_v for violet we infer that all bright bands, excepting the central bright band, will be coloured in which red will be in the outermost position while violet will be in the innermost position. When the path difference between interfering waves is large the condition for constructive interference for one wavelength and the condition for destructive interference for another wavelength may be satisfied at the same point. In that case the resultant illumination cannot be distinguished from white light. So for observable fringes with white light the path difference should be kept very small. In this case we get a few coloured fringes on either side of the central white fringe.

9.5 CONDITIONS FOR OBSERVABLE INTERFERENCE PATTERN :

In order to have well defined observable interference pattern the following conditions must be satisfied.

(i) *The two beams of light which interfere must be coherent* (See Art. 18.2).

Two sources are said to be coherent if the phase difference ϕ between the sources remains constant in time. If the sources are incoherent ϕ changes continually and we get uniform general illumination.

(ii) *The interfering waves must have the same frequency*. Also, their *amplitudes must be equal or very nearly equal.*

If the amplitudes a_1 and a_2 differ widely then the intensity, $(a_1 + a_2)^2$, in the bright region and that, $(a_1 \sim a_2)^2$ in the dark region will not differ significantly and hence intensity variation cannot be recognised.

(iii) *The original source must be monochromatic or very nearly monochromatic. If the light source is heterogeneous the optical path difference between the interfering beams must be kept very small.*

The spacing $\lambda D/d$ between consecutive bright or dark fringes is a function of λ. So fringes for different colours will be in step only at the central fringe and soon get out of step on either side of central fringe. If the path difference is large then dark fringes for some wavelengths may be masked by the bright fringes of some other wavelengths.

(iv) *The two interfering beams must propagate along the same direction or must intersect at a very small angle.*

If the angle between the two interfering wavefronts is large or the distance between the coherent sources is large the spacing between the interference fringes becomes small and may become indistinguishable even under high magnification.

(v) *For interference with polarised light the waves must be in the same state of polarisation.*

9.6 TWO CLASSES OF INTERFERENCE :

Optical devices producing interference fringes may be classified into the following two classes. The basis of classification depends on how we produce coherent sources.

(i) Division of wavefront :

Optical devices which divide the incident wavefront into two parts by reflection, refraction or diffraction and thereby give rise to two coherent interfering beams come under the division of wavefront class. In order to maintain spatial coherence it is essential to use narrow sources in these cases. The formation of fringes by Biprism, Lloyd's single mirror, Billet's divided lens etc. belong to this category. Since limited portions of the wavefront are used in these devices, diffraction effects are also present along with the interference effects.

(ii) Division of amplitude :

Optical devices which divide the amplitude of incident light wave into two or more parts by partial reflection and refraction and thereby give rise to two or more coherent interfering beams of light come under the division of amplitude class. Here we require to use broad source of light. As the interference effects corresponding to different points of the source are superposed here we get brighter bands. Since a large section of the wavefront is used diffraction effects are minimised. The formation of fringes by thin films, Newton's ring, Michelson's interferometer, Fabry-Perot interferometer etc. belong to this category.

9.7 YOUNG'S EXPERIMENT :

The phenomenon of interference was first demonstrated by Thomas Young in 1801 with a simple experiment. He allowed sunlight to fall on a pinhole S and then at some distance away on two pinholes S_1 and S_2 as shown in Fig. 9.7-1. Finally the light was received on an opaque screen. He observed that the illumination on the screen consisted of many alternate bright and dark spots.

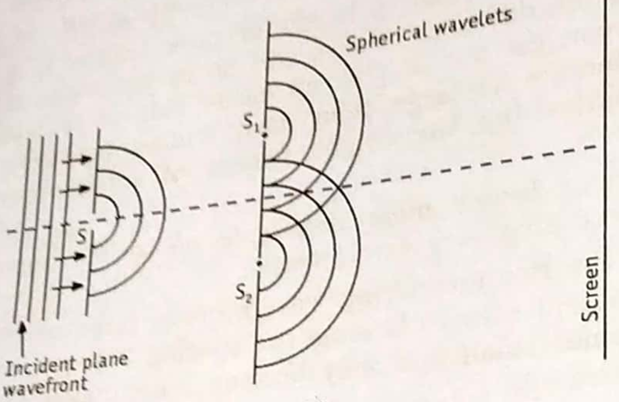


Fig. 9.7-1

According to Huygens' principle spherical wavelets spread out from the pinhole S. Since $SS_1 = SS_2$, these wavelets reach S_1 and S_2 at the same instant of time. Hence the new spherical wavelets diverging from S_1 and S_2 to the right will have equal phase at the start. Thus they act as two coherent sources. The two sets of spherical waves emerging from S_1 and S_2 interfere with each other and form a symmetrical pattern of bright and dark regions on the screen. At points where the waves meet in same phase maximum brightness is produced. On the other hand, minimum brightness is produced at points where the waves meet in opposite phase.

In modern version of the experiment the pinholes are replaced by narrow parallel slits.

9.8 FRESNEL'S BIPRISM :

Fresnel used a biprism to demonstrate the interference phenomenon. A biprism is essentially two prisms each of very small refracting angle (~30′) placed base to base. In practice it is constructed from a single glass plate. Experimental arrangement using Fresnel's biprism is shown schematically in Fig. 9.8-1. Light from a narrow slit S, illuminated by monochromatic light is incident symmetrically on the biprism‡ LMN. The incident wavefront is divided into two parts and suffer separate refractions through the two halves of the biprism. The two refracted wavefronts appear to diverge from two virtual sources S_1 and S_2. Hence

‡ It is desirable to place the plane surface of the biprism towards the source, for in that case deviation of the ray produced by the two surfaces of the prism will be nearly equal and hence the width of the virtual sources S_1 and S_2 would be small which will make the fringes distinct.

S_1 and S_2 can be considered as two coherent sources. The emergent wavefronts meet at small angles and produce interference pattern on the screen in the overlapping region PQ. The brightness or darkness

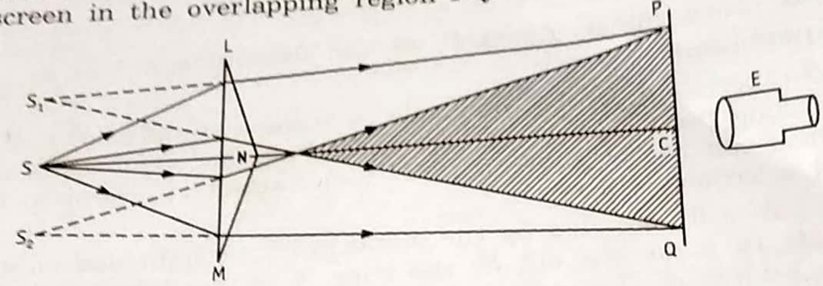
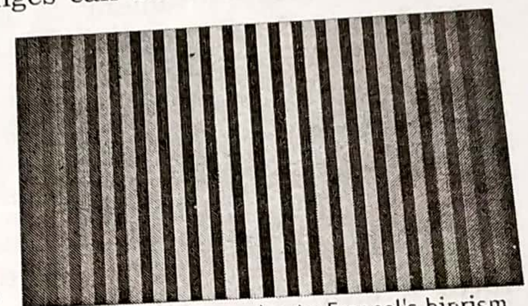


Fig. 9.8-1

at a point on the screen will be dependent only on the path difference of the point from S_1 and S_2. These fringes can be seen by an eye-piece E having a cross-wire. Typical fringe pattern is shown in Fig. 9.8-1(a).



Interference fringe due to Fresnel's biprism
Fig. 9.8-1(a)

Necessity of narrow source :

A broad source of light is equivalent to a large number of narrow sources placed side by side. Now if the slit is broad the two virtual coherent sources will also be broad. Now each pair of conjugate points on the virtual sources will give rise to an interference pattern. These interference patterns are slightly displaced from one another. An overlapping of such patterns results in general illumination.

Fresnel's biprism can be used for various optical measurements as discussed below.

(a) Determination of unknown wavelength :

Fresnel's biprism can be used to determine the wavelength of monochromatic light.

Theory :

Fringe width β is given by (*for derivation see Art.* 9.3)

$$\beta = \frac{\lambda D}{d} \qquad \text{...(9.8-1)}$$

where, D = distance between the slit and the screen.

d = distance between the two virtual coherent sources S_1 and S_2.

Therefore, unknown wavelength

$$\lambda = \frac{d\beta}{D} \qquad ...(9.8\text{-}2)$$

Thus measuring β, d and D we can determine λ.

Experiment :

The experiment can be conducted by using a suitable form of optical bench along the bed of which can slide a number of uprights carrying the linear slit S, the biprism LMN and Ramsden's eye-piece E fitted with a micrometer screw.

The slit is illuminated by the monochromatic light and adjustments are made to make the slit S, the edge N of the biprism and one of the cross-wires of the eye-piece perfectly vertical and all in the same vertical plane and at the same height from the bench. At this time the fringes will be very distinct. The biprism and eye-piece stands are given proper lateral movement so that fringes do not suffer lateral shift relative to the corss-wire as the eye-piece is moved.

The fringe width β is now measured by setting the cross-wire at successive fringes with the help of micrometer screw fitted with the eye-piece. Distance D can be measured directly from the bench scale as the distance between the slit and the eye-piece.

To measure d a convex lens of suitable focal length is placed on another upright inserted between the biprism and the eye-piece. The focal length of the convex lens is such that the distance between the slit and the eye-piece is greater than four times the focal length of the lens. Under this condition, there are two conjugate positions of the convex lens for which real images of S_1 and S_2 will be seen by the eye-piece kept at the same place. The distances d_1 and d_2 between the real images of S_1 and S_2, for the first and second positions of the convex lens respectively, are measured by moving the eye-piece perpendicular to the bench. Now magnification at one position will be inverse of magnification at the second position i.e.,

$$\frac{d_1}{d} = \frac{d}{d_2} \quad \text{or,} \quad d = \sqrt{d_1 d_2}$$

There may be index error between slit stand and eye-piece stand. It can be corrected for or avoided by measuring fringe widths β_1 and β_2 at two different distances D_1 and D_2 respectively. Then, instead of Eq. (9.8-2), λ is given by

$$\lambda = d\,\frac{\beta_2 - \beta_1}{D_2 - D_1} \qquad ...(9.8\text{-}3)$$

As all the quantities on the right hand are known, λ can be easily determined.

(b) Measurement of the acute angle of biprism :

From Fig. 9.8-1, it is evident that the deviation of each of the rays SL and SM after their passage through the two thin prisms having acute angles at L and M respectively, is given by $\delta = \angle SLS_1 = \angle SMS_2$.

The angular separation of the two virtual sources S_1 and S_2 will then be equal to 2δ (in radian). But $\delta = (n-1)\alpha$, where α is the acute angle at L or at M of the two thin prisms. If a is the distance between the slit S and the biprism, then $2\delta = d/a$ where d is the linear distance between the two virtual sources S_1 and S_2 [$\because$ d is very small],

$$2(n-1)\alpha = \frac{d}{a}; \text{ or, } d = 2a\,(n-1)\alpha \qquad \ldots(9.8\text{-}4)$$

or,

Measuring d and a, and knowing n the refractive index of the material of prism for the monochromatic light employed, we can find α. The distance a between the slit and biprism can be obtained directly from the bench scale. The distance 'd' between virtual sources can be measured as described earlier. *The base angle α of the biprism is kept small*. If it is made large then the distance d between the virtual sources becomes large and fringe width β becomes small. For large α, fringe width may become so small that it cannot be distinguished even under high magnification.

(c) Measurement of the thickness of a thin film :

Fresnel's biprism can be used to measure the thickness of a given thin sheet of transparent material.

Let S_1 and S_2 be the two virtual coherent sources which are producing interference fringes on the screen so that C is the position of the central bright band of zero optical path difference i.e., $S_1C = S_2C$. If a thin film of thickness t be introduced into one of the paths (say, S_1P) of the interfering rays, then the position of the central fringe will be shifted from C to P (say), so that the optical path S_1P is again equal to the optical path S_2P. The time taken by light in going from S_1 to P and from S_2 to P will be equal. Thus,

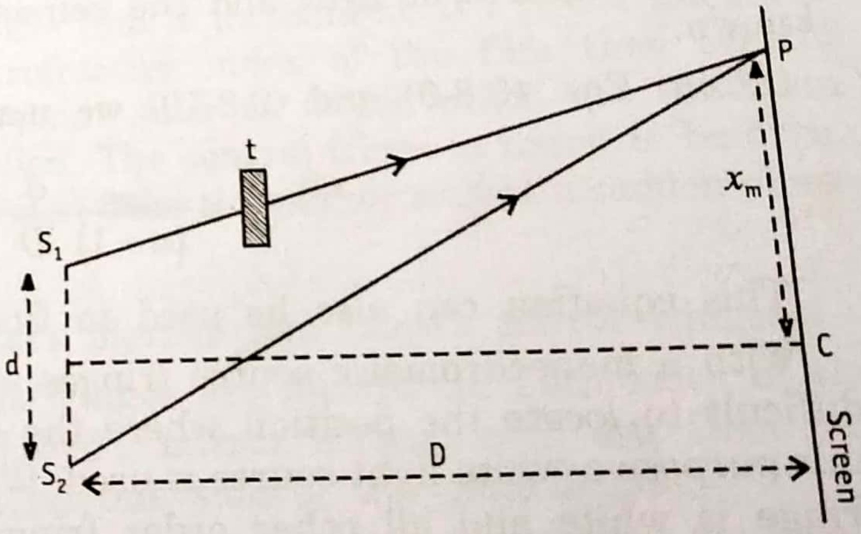


Fig. 9.8-2

$$\frac{S_2P}{c} = \frac{S_1P - t}{c} + \frac{t}{v}$$

or,
$$S_2P - S_1P = (n-1)t \qquad ...(9.8\text{-}5)$$

where c and v are the velocity of light in air and in the film; $n = c/v$ is the refractive index of the material of the film.

If P is the position originally occupied by the mth order fringe then

$$S_2P - S_1P = m.\lambda \qquad ...(9.8\text{-}6)$$

Therefore, from Eqs. (9.8-5) and (9.8-6) we get,

$$(n-1)t = m\lambda \qquad ...(9.8\text{-}7)$$

The lateral shift of the central fringe of zero optical path difference is given as

$$x_m = CP = m\beta \qquad ...(9.8\text{-}8)$$

where $\beta = \dfrac{\lambda D}{d}$ is the fringe width ...(9.8-9)

From Eqs. (9.8-7) and (9.8-8) we get

$$(n-1)t = \frac{x_m}{\beta}.\lambda$$

or,
$$t = \frac{x_m.\lambda}{\beta(n-1)} \qquad ...(9.8\text{-}10)$$

Finding x_m, the displacement of the central fringe due to introduction of the thin film and β, the distance between two consecutive bright bands, we can find t, the thickness of the film by Eq. (9.8-10), when the wavelength λ of light and the refractive index n of the film are known.

From Eqs. (9.8-9) and (9.8-10) we may write,

$$t = \frac{x_m}{(n-1)}\cdot\frac{d}{D} \qquad ...(9.8\text{-}11)$$

This equation can also be used to find t.

With a monochromatic source fringes appear to be similar and it is difficult to locate the position where the central fringe is shifted. For this purpose a white light source is used. With a white source the central fringe is white and all other order fringes are coloured.

Velocity test :

The relation (9.8-11) may be written as

$$x_m = \frac{D}{d}\left(\frac{c}{v} - 1\right)t \qquad ...(9.8\text{-}12)$$

If the light travels with smaller velocity in the denser medium (film) then c/v is greater than 1 and x_m is positive. Hence the central fringe will be shifted towards the side of the film. Experimental result agrees with this inference arrived at. Hence the conclusion is that light travels with smaller velocity in denser medium.

9.9 LLOYD'S SINGLE MIRROR :

In Lloyd's single mirror arrangement (Fig. 9.9-1) light from a narrow slit S_1 illuminated with a monochromatic light is partly incident at a grazing angle on a metallic mirror M_1M_2 while the rest reaches the screen AB directly. The reflected light appears to diverge from a virtual source S_2. Hence S_1 and S_2 act as coherent sources and interference fringes are formed in the region of overlapping EF.

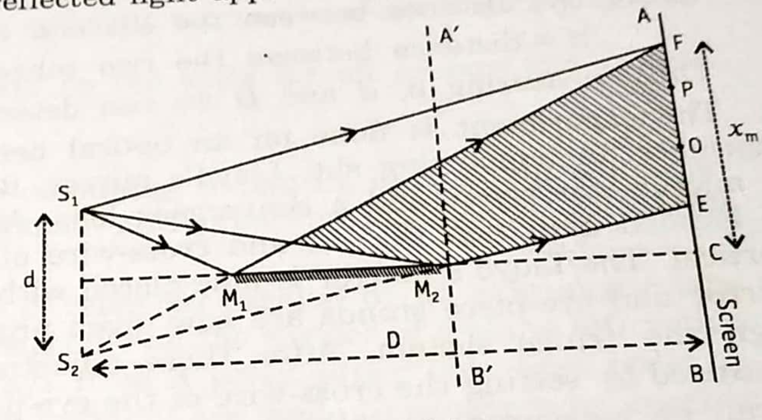


Fig. 9.9-1

The central point C on the screen for which $CS_1 = CS_2$ receives only the direct light and for this the central fringe of zero path difference is not visible here. However, if the screen is displaced to the position $A'B'$ the central fringe can be brought into view. The central fringe can also be brought into view by introducing a thin film of mica or glass in the path of direct light. In this case the fringe system shifts in direction in which the film is introduced. If t is the thickness of the film and n is the refractive index of the film then central fringe will be formed at a point such as O for which the condition $S_2O = S_1O + (n-1)t$ is satisfied. The central fringe is found to be dark. This indicates that light reflected from the mirror suffers a sudden phase change of π.

Difference between Fresnel's biprism and Lloyd's mirror fringes :

(i) In biprism experiment fringes are formed on both sides of the central fringe, whereas in Lloyd's mirror arrangement less than half of the fringes are obtained on one side of the central line.

(ii) In biprism the central fringe is bright whereas in Lloyd's mirror it is dark.

(iii) In Fresnel's biprism the separation (d) between every pair of corresponding points of the coherent sources is same and hence fringe width is same for all parts of the source. But in Lloyd's mirror due to lateral inversion in the mirror d is different for different pair of coherent point sources. Consequently in Lloyd's mirror fringe width is not same for all parts of the source slit.

Measurement of wavelength :

Lloyd's mirror can be used to measure the wavelength (λ) of a monochromatic light.

Fringe width β is given by (for deduction see Art. 9.3)

$$\beta = \frac{\lambda D}{d} \; ; \text{ or, } \; \lambda = \frac{d.\beta}{D} \qquad \text{...(9.9-1)}$$

where D = distance between the slit and the screen.

d = distance between the two coherent sources.

Thus measuring β, d and D we can determine λ.

The experiment is done on an optical bench fitted with movable uprights for supporting slit, Lloyd's mirror, Ramsden's eye-piece with a micrometer screw and a convergent lens. At first the optical bench is levelled and then the slit and cross-wire of the eye-piece are made vertical. The Lloyd's morror is now placed with its surface vertical. The mirror and eye-piece stands are now given proper lateral movement to align the fringe system. After these adjustments fringe width β is measured by setting the cross-wire of the eye-piece at successive fringes. D can be measured directly from the bench scale as the distance of the eye-piece from the slit. To find d a convex lens of suitable focal length (f) is placed between the Lloyd's mirror and the eye-piece. If the eye-piece is kept at a distance greater than $4f$ from the slit, we get two distinct real images of the coherent sources on the eye-piece for two conugate positions of the lens. Let d_1 and d_2 be the distances between the real images for these two positions of the lens. Now magnification at one position will be inverse of magnification at the second position i.e.,

$$\frac{d_1}{d} = \frac{d}{d_2} \; ; \text{ or, } \; d = \sqrt{d_1 d_2} \qquad \text{...(9.9-2)}$$

Thus measuring β, D and d we can find λ from Eq. (9.9-1).

There may be index error between the slit stand and eye-piece stand. It can be corrected for or avoided by measuring β for two values of D. Then instead of Eq. (9.9-1) one uses the relation

$$\beta = d \cdot \frac{\beta_2 - \beta_1}{D_2 - D_1} \qquad \text{...(9.9-3)}$$

one another and hence the fringes are not very sharp unless the slit is made very narrow.

(8) Billet's divided lens method :

In this arrangement two real coherent sources are obtained from a single source S. In Fig. 9.11.8 L_1 and L_2 are the two halves of a split convex lens, which form two real images S_1 and S_2 of a single linear source S illuminated by monochromatic light. The beams of light which diverge from the real coherent sources S_1 and S_2 superpose on the screen

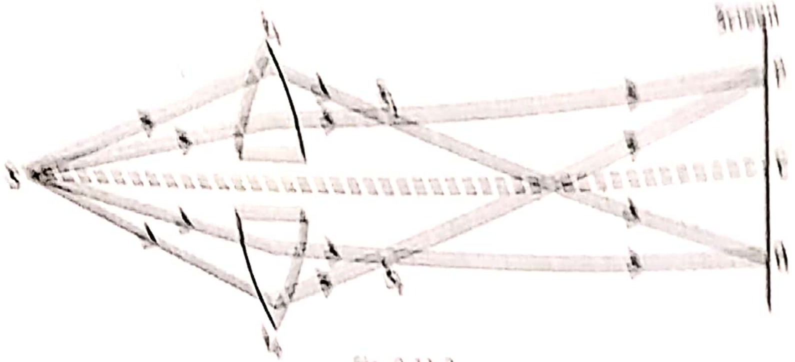


Fig. 9.11.8

within the region PQ where interference fringes will be produced having central bright band at C which is equidistant from S_1 and S_2.

9.12 PHASE CHANGE ON REFLECTION : STOKE'S TREATMENT :

Suppose a light wave of amplitude a is incident on an interface of two media as shown in Fig. 9.12.1. Let r and t represent the fractions of the amplitude of the incident wave which are respectively reflected above and refracted below the surface. Hence the amplitudes of the reflected and refracted waves for an incident wave of amplitude a, will be ar and at respectively.

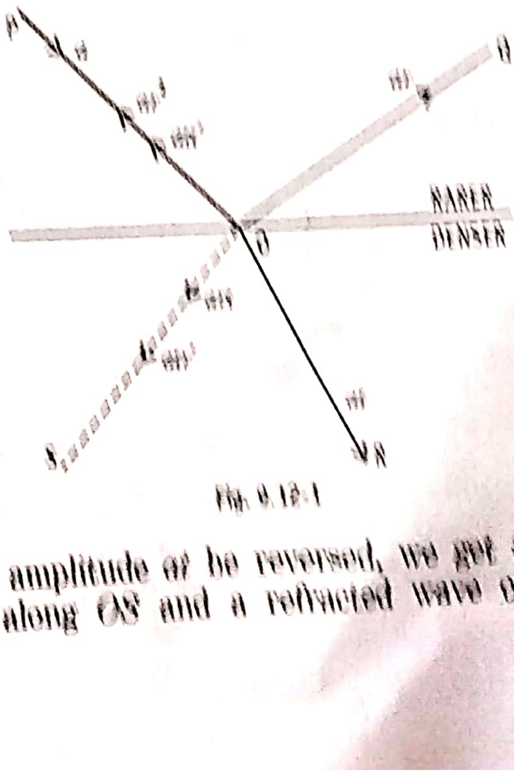


Fig. 9.12.1

If there is no absorption of energy, the wave motion is a strictly reversible phenomenon. Now if the reflected wave along OQ having the amplitude ar be reversed, we get one reflected wave of amplitide ar^2 along OP and another refracted wave of amplitude art along OS. Similarly if the wave along OR having the amplitude at be reversed, we get a reflected wave of amplitude atr' along OS and a refracted wave of amplitude att' along OP where r' and t' represent respectively the amplitude reflection and transmission coefficients when the wave is incident from lower medium. Since the resultant effect must consist of a wave along OP of amplitude a we have,

$$att' + ar^2 = a\ ; \quad art + atr' = 0 \qquad \ldots(9.12\text{-}1)$$

From these equations we get,

$$tt' = 1 - r^2 \qquad \ldots(9.12\text{-}2)$$

$$r' = -r \qquad \ldots(9.12\text{-}3)$$

Relation (9.12-3) indicates a phase change of π between reflections in the denser and rarer media. Lloyd's mirror experiment shows a phase change of π on reflection from a surface backed by a denser medium. Hence we conclude that no such abrupt phase change occurs when light is reflected by a surface backed by rarer medium.

9.13 INTERFERENCE PHENOMENA IN THIN FILMS, BASED ON THE DIVISION OF AMPLITUDE :

Beautiful colours are often exhibited when a beam of white light from an extended source is reflected from a thin film of oil floating on water or from a soap bubble. These phenomena can be explained on the basis of interference of light waves reflected from the upper and the lower surfaces of the film.

(a) Interference in reflected light :

Let AB and CD be the bounding surfaces of a thin wedge-shaped film enclosing an angle α (Fig. 9.13-1).

Suppose a ray PQ of monochromatic light of wavelength λ is incident on the film. This ray will be partly reflected along QO from the front surface and partly refracted along QR. The ray along QR after being partly reflected from the back surface CD, emerges along SO and meets the reflected ray QO at O.

The rays QO and SO are derived from the same incident ray PQ and hence are coherent. They combine to produce interference phenomena. Let us now calculate the phase difference between these reflected beams. Draw $SN \perp^r QO$, $SN_1 \perp^r QR$ and $SLM \perp^r CD$. Produce QR and SL to meet at M. The paths of the two reflected beams (QO and SO) which are going to meet at O, will be equal from the dotted line SN upto O (for they are very close to each other). Hence the path difference of the two reflected interfering beams would be,

$$l = n(QN_1 + N_1R + RS) - QN \qquad \ldots(9.13\text{-}1)$$

where n = refractive index of the film.

From the geometry of Fig. 9.13-1, $RS = RM$ and $SL = LM = d$ = hickness of the film at S. By Snell's law we get,

$$n = \frac{\sin i}{\sin r} = \frac{QN/QS}{QN_1/QS}$$

or,
$$QN = nQN_1$$

Thus Eq. (9.13-1) now reduces to,

$$l = n(N_1R + RS) = n\,(N_1R + RM) = nN_1M$$

or,
$$l = 2nd\cos(r - \alpha) \qquad \text{...(9.13-2)}$$

In addition to this path difference, there is an extra phase difference of π equivalent to a path difference $\pm\lambda/2$, caused by reflection at Q,

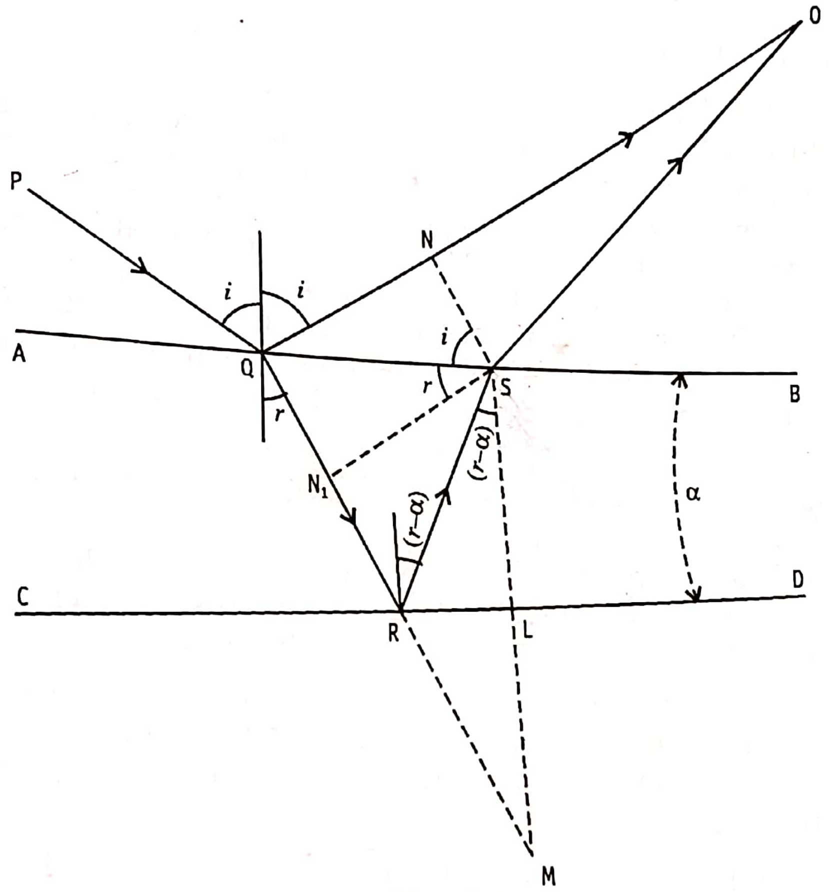


Fig. 9.13-1

from the surface backed by denser medium. The reflection at R, from the surface backed by rarer medium, will not cause any change of phase.

Hence the total path difference between the two reflected rays is,

$$l = 2nd\cos(r-\alpha) \pm \frac{\lambda}{2} \qquad \text{...(9.13-3)}$$

Conditions for maxima and minima—

For maxima of brightness at O,

$$2nd\cos(r-\alpha) \pm \frac{\lambda}{2} = \text{even multiples of } \frac{\lambda}{2}$$

or,
$$2nd\cos(r-\alpha) = \text{odd multiples of } \frac{\lambda}{2}$$
$$= (2m+1)\frac{\lambda}{2}$$

where $m = 0, 1, 2, 3...$ ...(9.13-4)

Similarly, for minima of brightness at O,

$$2nd\cos(r-\alpha) = 2m\frac{\lambda}{2}$$ ...(9.13-5)

Note that for a parallel film $\alpha = 0$.

(a) Fringes with monochromatic light :

(i) When a parallel beam of monochromatic light is incident on a film of *varying thickness*, n, λ and r are all constants. Hence different order numbers (m) of the fringes will be obtained for different thicknesses (d) of the film. At the thin edge of the film, where d is practically zero, the path difference (= $2nd \cos r$) of the two reflected rays inside the film is zero. The only path difference between these two reflected rays is $\lambda/2$, caused by reflection from the surface AB of the film. Hence this thin edge would be dark not only for a light of one colour but for lights of all colours.

As the thickness, d of the film increases, the order number m of the fringe also increases. Thus a fringe of a given order number will be the locus of the points where film thickness d is remaining same. If the film surfaces are perfectly plane, we get straight fringes parallel to the line of intersection of the surfaces of the film.

(ii) When the film is *extremely thin*, so that d is practically zero for all parts of the film, then the only path difference between the two reflected rays is $\lambda/2$ and the film surface will appear perfectly dark even with white light.

(iii) If the film surfaces are *plane-parallel* (i.e., $\alpha = 0$) and a parallel beam of monochromatic light is incident on its surface obliquely, then n, λ and d are all constants but r will vary with the inclination of the parallel beam with the surface. Hence the film surface will be uniformly bright or dark according as the value of r satisfies the conditions of brightness and darkness given in the relations (9.13-4) and (9.13-5) respectively. If the parallel beam is incident on such a film normally then n, d and λ are all constants and $\cos r = 1$. Hence the film surface will be uniformly bright or dark according as the thickness d satisfies the conditions of brightness and darkness.

(b) Fringes with white light-colours of thin film :

When a parallel beam of white light is incident on a thin wedge-shaped film, the values of n, λ and r will be different for lights of different colours. At the thinner edge of the film, thickness (d) is practically zero and this edge will be perfectly dark for light of all colours owing to the introduction of path difference of $\lambda/2$ by reflection from the surface AB of the film. As $\lambda_v < \lambda_r$, the first order bright fringe of violet light will be formed at a smaller thickness of the film while the corresponding bright fringe of red light will be formed at the greater thickness. Thus we get differently coloured fringes at different thicknesses and these fringes are called *fringes of equal chromatic order.*

Beyond this point, we get another thickness where the condition of brightness will be simultaneously satisfied by two or more colours and here we get a coloured band due to the overlapping of these coloured bright fringes. When the thickness of the film is sufficient, the overlapping of differently coloured bright fringes goes to such an extent that we get uniform illumination. Thus we explain the colouration of thin films.

(b) Interference in transmitted light :

A similar interference phenomenon is seen to occur with the rays transmitted through the film and emerging on the side CD of it (Fig. 9.13-2). One part of the ray QR incident on the boundary CD, will emerge out of CD in the direction RO_1. Another part of it will also emerge out of CD along TO_1 after two successive internal reflections at R and S. These two internal reflections will cause no change of phase and hence the phase difference of these two emergent rays will be purely determined by their path difference.

In figure $NO_1 \approx TO_1$ and hence the path difference of the two transmitted interfering beams would be,

$$l = n(RN_1 + N_1S + ST) - RN \qquad ...(9.13\text{-}6)$$

Now, TN_1, TN and TLM are drawn perpendiculars on RS, RO_1 and AB respectively. RS and TL are produced to meet at M. From the geometry of the Fig. 9.13-2 we get, $ST = SM$ and $LT = LM = d$ = thickness of the film at L.

By Snell's law we get, $n = \dfrac{\sin i_1}{\sin(r - \alpha)} = \dfrac{RN/RT}{RN_1/RT}$

or, $$RN = nRN_1.$$

Thus Eq. (9.13-6) reduces to,

$$l = n(N_1S + SM) = nN_1M = 2nd\cos(r - 2\alpha) \qquad ...(9.13\text{-}7)$$

$\therefore$ For maxima of brightness,

$$2nd\cos(r-2\alpha) = 2m\frac{\lambda}{2} = m\lambda \qquad \ldots(9.13\text{-}8)$$

For minima of brightness,

$$2nd\cos(r-2\alpha) = (2m+1)\frac{\lambda}{2} \qquad \ldots(9.13\text{-}9)$$

For a film of very small wedge angle α or for a parallel film (α = 0), then we note from the Eqs (9.13-4), (9.13-5), (9.13-8) and (9.13-9) that the condition for maxima in the transmitted pattern corresponds to the condition for minima in the reflected pattern and vice-versa. Thus we

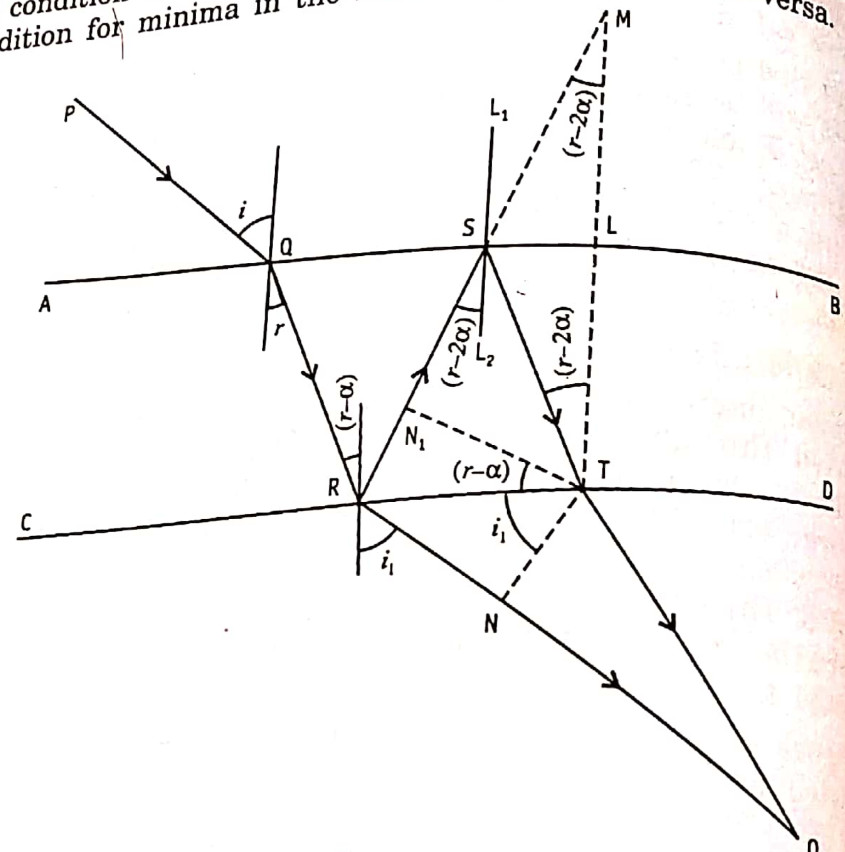


Fig. 9.13-2

may conclude that *the fringes observed with the reflected and transmitted lights are complementary to each other.*

Necessity of using a broad source and thin film :

If rays from a point source *P* [Fig. 9.13-3(a)] be made incident on the film then the reflected or transmitted rays that can enter the eye (whose aperture is small) are confined to a small range of directions. Hence field of view will be very small. On the other hand, if a broad source of light is used, light from every point of the source gives rise to a pair of coherent waves which can reach the eye after being reflected or transmitted from different points of the film. Thus the field of view become wide and fringes can be seen over the entire film [Fig. 9.13-3(b)].

For a thick film, the two adjacent reflected or transmitted rays will be wide apart and will not be able to enter the eye whose pupil is small.

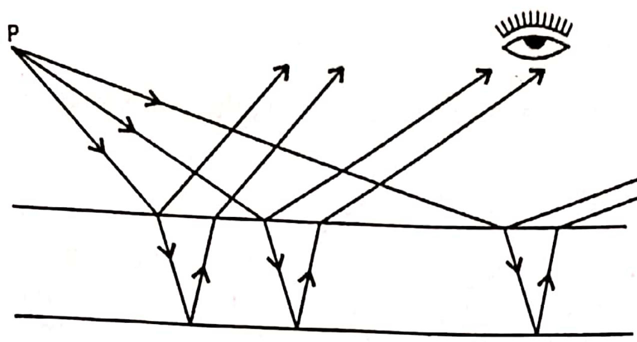


Fig. 9.13-3(a)

If the film be thin, then the two adjacent reflected or transmitted rays will be able to enter the eye where interference will occur by overlapping.

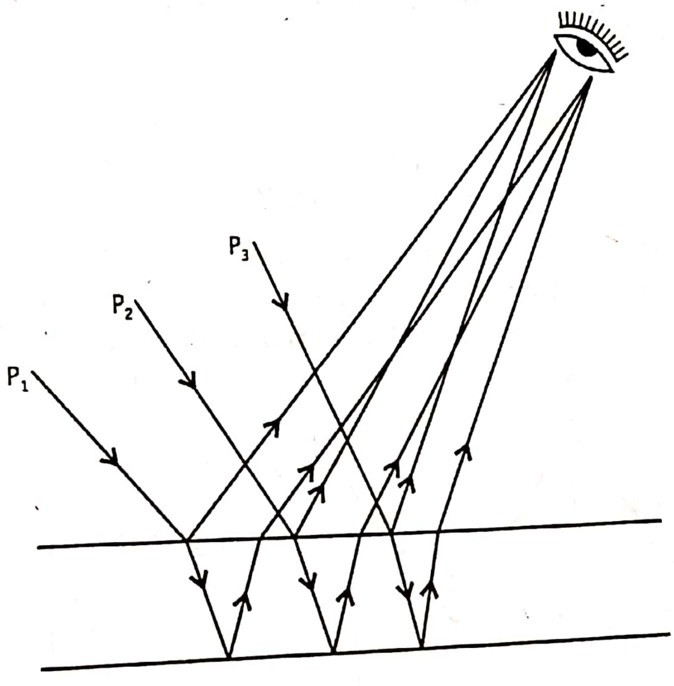


Fig. 9.13-3(b)

Fringe Width :

When a parallel beam of monochromatic light is incident normally on the film, the condition of brightness with reflected light is given by,

$$2nd = (2m+1)\lambda/2.$$

But the thickness d of the film at S is αx_m, where x_m is distance of the mth bright band from the apex of the film. Thus

$$2n\alpha x_m = (2m+1)\lambda/2.$$

$$x_m = \frac{(2m+1)\lambda}{4n\alpha}$$

or,

$$x_{m+1} = \frac{(2m+2+1)\lambda}{4n\alpha}$$

Similarly,

The distance (β) between two consecutive bright bands is given by,

$$\beta = x_{m+1} - x_m = \frac{\lambda}{2n\alpha} \quad ...(9.13\text{-}10)$$

Thus for a given film and for a light of given wavelength λ, β is constant. Hence the fringes are equi-spaced. When α increases the fringes become narrower but when α decreases the fringes become broader.

Fringes of equal width and equal inclination :

The conditions for maxima and minima of brightness of the fringes formed by the light reflected from a thin wedge-shaped film are respectively given by,

$$2nd\cos(r-\alpha) = (2m+1)\frac{\lambda}{2} \text{ (maxima)} \quad ...(9.13\text{-}11)$$

$$2nd\cos(r-\alpha) = 2m\lambda/2 \text{ (minima)} \quad ...(9.13\text{-}12)$$

The above conditions suggest the existence of two distinct types of fringes. Suppose the incident rays are parallel and monochromatic and hence n, r and λ are all constants. Under this condition different order numbers (m) of the fringes will be controlled by the thickness d of the film. Hence a fringe of a particular order number will lie on the locus of all the points of the film having a constant thickness. These fringes are called *fringes of equal width or thickness*. If the film surfaces are perfectly plane we get straight fringes which are all parallel to the line of intersection of the film surfaces, where central dark fringe is situated.

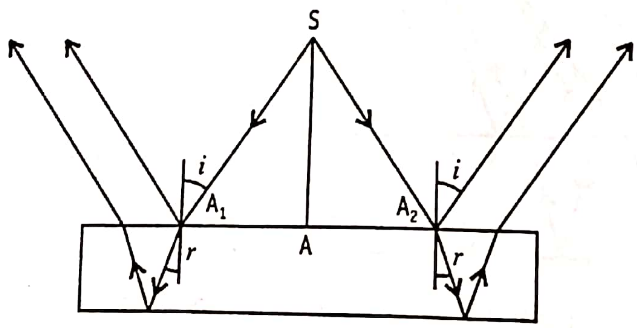


Fig. 9.13-4

Another type of fringes will be obtained when monochromatic divergent rays from an *extended source* are incident on a film of uniform thickness. Here different order numbers (m) of the fringes will be controlled by the different values of r. If we draw a circle, with the cutting point A of the normal ray SA with the surface as centre, and AA_1 as radius, then all rays incident on the circumference of this circle will have the same angle of incidence i (Fig. 9.13-4). Thus the fringe of a particular order will be a circle with the point A as centre. Fringe system will consist of concentric bright and dark rings. Fringe of a particular order is characterised by a particular angle of incidence and hence these fringes are called *fringes of equal inclination*. As the angle of incidence of the rays increases, the value of $\cos r$ decreases causing a decrease in the value of m. Thus the fringes of bigger radii will have smaller order number.

The Newton's rings (See Art. 9.16) are the example of fringes of equal width type. Here equal thickness of air film exists over the circumference of a circle, having the point of contact of convex lens and glass plate as centre and hence the fringes assume circular form.

The fringes of equal thickness are employed to test the optical planeness of a surface. For this purpose an air film is formed between the working surface and a standard optically flat surface. The fringes of equal thickness formed by the air film are repeatedly observed with a monochromatic light and the polishing of the working surface is continued until the fringes are perfectly straight and parallel to the line of intersection of the surfaces of the air film.

Fringes of equal inclination can be produced by transmitted light from a thick transparent plate (Fig. 9.13-5). As the plate is thick, the pair of adjacent transmitted rays will be wide apart and hence they cannot enter the eye through its small pupil to produce any interference phenomena there. But if a telescope with a bigger diameter objective be employed, then the objective will be able to collect those transmitted rays which are very close to the normal ray and consequently those collected rays will produce interference phenomena at the focal plane of the objective. The fringe pattern obtained in the focal plane of the telescope consists of concentric bright and dark rings. They are fringes of equal inclination type. These fringes are called **Haidinger's fringes.**

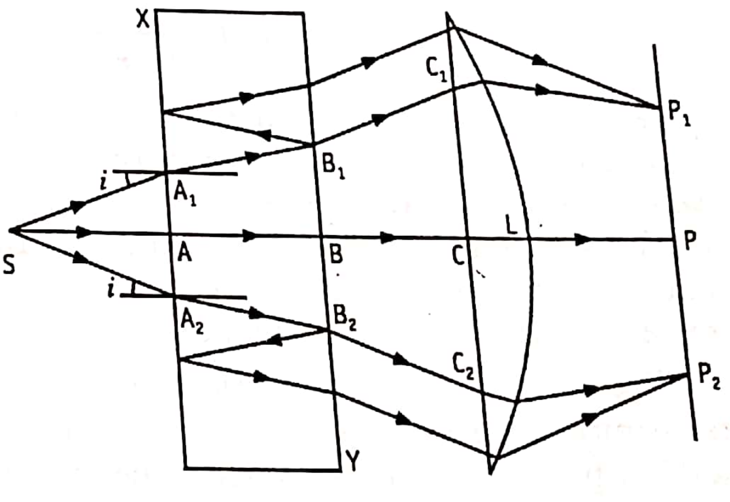


Fig. 9.13-5

Haidinger's fringes are employed to test the flatness of a plate to a high degree of accuracy. For accurately plane-parallel surfaces Haidinger's fringes will be perfectly circular but any deviation from the parallelism of the surfaces will be indicated by the distortion in the rings.

9.14 INFINITELY THIN FILM :

Let us consider interference of light waves reflected from the upper and lower surfaces of a very thin plane parallel film as indicated in Fig. 9.14-1. If the film thickness $d << \lambda$ then the geometrical path difference ($2nd \cos r$) between two consecutive reflected rays QR and Q_1R_1 becomes negligible and total phase difference between them equals π which arises due to reflection from the front surface of the denser film. So QR and Q_1R_1 will produce destructive interference. Since their amplitudes are different there will be no complete destruction. However, if we consider the effect of other waves produced by multiple internal reflections then we can show that there will be complete destruction.

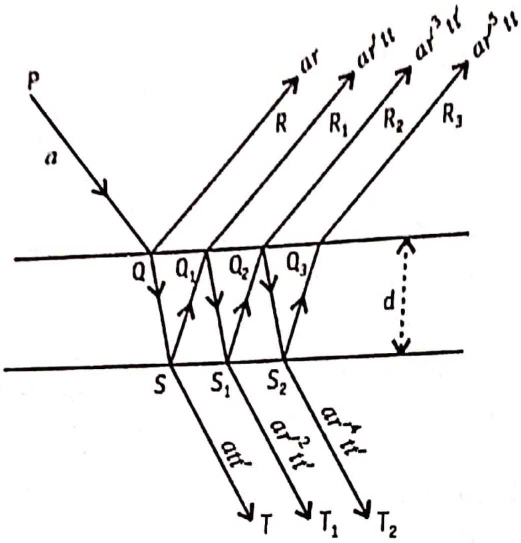


Fig. 9.14-1

The resultant amplitude of the waves reflected along Q_1R_1, Q_2R_2, Q_3R_3 and so on will be equal to

$$A = ar'tt'\left(1 + r'^2 + r'^4 + \ldots \text{upto } \infty\right)$$

$$= ar'tt' \cdot \frac{1}{1 - r'^2}$$

Now from Stoke's treatment $tt' = 1 - r^2$ and $r' = -r$. Therefore, $A = -ar$. So the overall reflected amplitude including QR is $ar - ar = 0$. Thus, when the film is very thin compared to wavelength, it appears *perfectly dark* when seen by reflected light. The film does not reflect any light and hence to satisfy the principle of conservation of energy all light must be transmitted through the film.

Since $d << \lambda$ the geometrical path difference between consecutive transmitted waves may be neglected. The transmitted waves suffer no sudden phase change and hence they are all in equal phase. Therefore, resultant amplitude of the transmitted waves is

$$A' = att' + att'r'^2 + att'r'^4 + \ldots\ldots$$

$$= att'\left(1 + r'^2 + r'^4 + \ldots\ldots\right)$$

$$= att' \cdot \frac{1}{1 - r'^2}$$

Since $tt' = 1 - r'^2$, $A' = a$ (the amplitude of incident wave). Thus we conclude that a very thin transparent film transmits whole of the incident light to the other side.

9.15 NON-REFLECTING FILM :

A part of the incident light is lost due to reflection at lens surfaces. Although the loss is small for a single surface, in a multi-lens system the loss is significant. Thus it is important to find ways of reducing or eliminating such losses. One way to eliminate reflection is to use a thin layer of transparent material on the lens surface. This is known as *anti-reflection coating*. The material and thickness of the film are to be chosen so that the two consecutive reflected rays QR and Q_1R_1, as shown in Fig. 9.15-1, are out of phase and their amplitudes are equal. Under these conditions the reflected waves undergo complete destructive interference. No light is then reflected and if the film is transparent absorption in it is negligible and whole of the incident light passes through the coating into the glass.

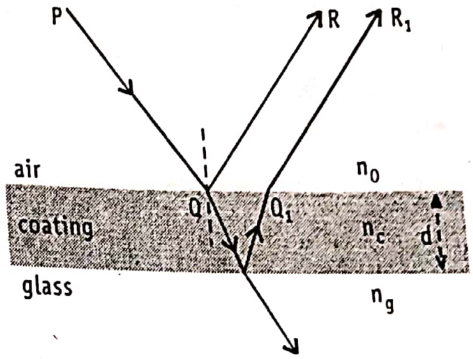


Fig. 9.15-1

According to the electromagnetic theory of light the *reflectivity* r at the boundary between two media of refractive indices n_1 and n_2 and at normal incidence is given by

$$r = \left(\frac{n_2 - n_1}{n_2 + n_1}\right)^2 \qquad ...(9.15\text{-}1)$$

So for equality of the amplitudes of the waves along QR and Q_1R_1 we have

$$\left(\frac{n_c - n_0}{n_c + n_0}\right)^2 = \left(\frac{n_g - n_c}{n_g + n_c}\right)^2 \qquad ...(9.15\text{-}2)$$

which, on simplification, shows that the refractive index of the coating must be

$$n_c = \sqrt{n_0 . n_g} \qquad ...(9.15\text{-}3)$$

Here both the rays QR and Q_1R_1 suffer sudden phase change of π and hence for destructive interference the geometrical path difference between them must be equal to $\lambda/2$. Thus for normal incidence the minimum thickness 'd' of the coating is given by

$$2n_c d = \lambda/2$$

or,
$$d = \frac{\lambda}{4n_c} \quad ...(9.15\text{-}4)$$

Thus a coating of refractive index as given by the Eq. (9.15-3) and of thickness as given by the Eq. (9.15-4) will serve as anti-reflection coating. The material of the coating must be chosen as to be insoluble, durable and scratch resistant. Most frequently used materials are magnesium fluoride and cryolite. The technique of reducing the reflectivity of a surface is known as *blooming*.

The anti-reflection coating using a single layer is effective for one wavelength or over a narrow range of wavelengths. It is possible to produce anti-reflecting coating that are efficient over a wide range of wavelengths by using multilayer coatings.

9.16 NEWTON'S RINGS :

Newton's rings are a particular example of interference fringes formed by thin films. By placing a plano-convex lens on a plane glass plate a thin air film of progressively increasing thickness from the point of contact O can be formed as shown in Fig. 9.16-1. If it is illuminated by a monochromatic light interference fringes in the form of concentric circular rings are found. These rings were first observed by Newton and are known as *Newton's rings*. These fringes are the loci of points of *equal film thickness*. These rings are *localized* in the air film. Typical Newton's rings are shown in Fig. 9.16-1(a).

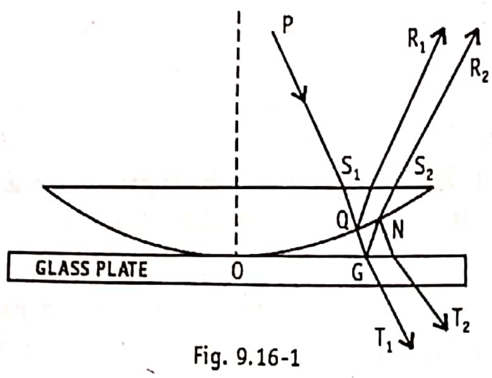
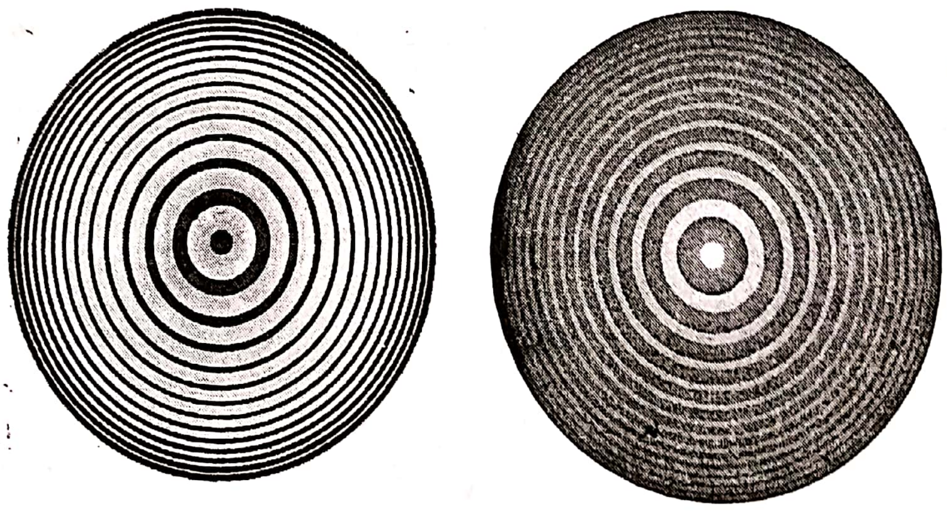


Fig. 9.16-1



Newtons rings in reflected light

Newtons rings in transmitted light

Fig. 9.16-1(a)

Theory :

An explanation to the formation of the rings can be given in terms of interference of light waves reflected from the convex lower surface of the lens and the flat upper surface of the glass plate. The lens is usually of large radius of curvature such that the thickness of the air film is very small. The experimental arrangement is so designed that light falls on the film almost normally. Under such circumstances the optical path difference between two successive reflected waves QS_1R_1 and NS_2R_2 as shown in Fig. 9.16-1, will be

$$2nd \pm \lambda/2 \qquad \text{...(9.16-1)}$$

where d = thickness of the film at N. Here we assume the film to be optically rarer than the media, above and below and consequently we are to consider the abrupt phase change of π due to reflection at G. This is equivalent to a path difference of $\pm \lambda/2$.

Therefore, the waves interfere constructively, if

$$2nd \pm \frac{\lambda}{2} = \text{even multiple of } \lambda/2$$

or,

$$2nd = \text{odd multiple of } \lambda/2$$
$$= (2m+1)\lambda/2 \qquad \text{...(9.16-2)}$$

where $m = 0, 1, 2, 3...$

For destructive interference,

$$2nd \pm \lambda/2 = \text{odd multiple of } \lambda/2$$

or,

$$2nd = \text{even multiple of } \lambda/2$$
$$= 2m.\lambda/2 \qquad \text{...(9.16-3)}$$

A fringe of a given order (m) will be along the loci of points of equal film thickness (d) and hence the fringes will be circular. The radii of the rings can be found out from Fig. 9.16-2. If the point Q fulfils the condition of brightness (or darkness) then all points on the circumference of a circle of radius Q_1Q will be bright (or dark). Thus we shall get a bright (or dark) ring, of say mth order, whose radius is $Q_1Q = r_m$. By geometry, $R^2 = r_m^2 + (R-d)^2$.

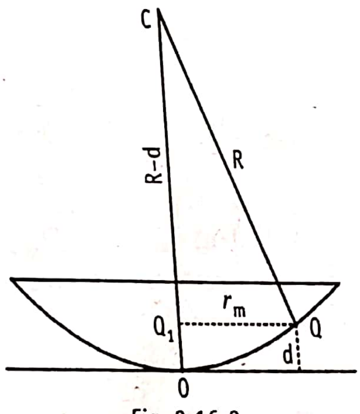


Fig. 9.16-2

where R = radius of curvature of the convex surface.

Since $R >> d$, we can write $r_m^2 \approx 2Rd$ ...(9.16-4)

Using the condition (9.16-2) we can write for the mth bright ring.

$$r_m^2 = \frac{(2m+1)\lambda R}{2n} \qquad \text{...(9.16-5)}$$

Similarly for the mth dark ring,

$$r_m^2 = \frac{2m.\lambda R}{2n}$$ ...(9.16-6)

Thus we find that the radii of the bright rings are proportional to the square root of odd natural numbers and that of the dark rings proportional to the square root of natural numbers.

Corresponding diameters are given by

$$D_m^2 = \frac{2(2m+1)\lambda R}{n} \quad \text{[Bright ring]}$$ ...(9.16-7)

$$D_m^2 = \frac{4m\lambda R}{n} \quad \text{[Dark ring]}$$ ...(9.16-8)

The difference in diameters of the mth and $(m+1)$th order dark rings is

$$D_{m+1} - D_m = \sqrt{\frac{4\lambda R}{n}}\left[\sqrt{m+1} - \sqrt{m}\right]$$

Thus as the order number 'm' increases this difference decreases meaning that the rings gradually become narrower as their radii increase. There will be more crowding of the rings as we move outward from the centre.

Fringe width :

If D_m and D_{m+1} are the diameters of two successive bright rings then we can write from Eq. (9.16-7),

$$D_{m+1}^2 - D_m^2 = \frac{4\lambda R}{n}$$

or,

$$D_{m+1} - D_m = \frac{4\lambda R}{n(D_{m+1} + D_m)}$$

Writing $D_{m+1} + D_m \approx 2D_m$, we get fringe width β as,

$$\beta = \frac{D_{m+1} - D_m}{2} = \frac{\lambda R}{n.D_m}$$ ...(9.16-9)

Thus fringe width β decreases as the diameter D_m of the ring increases.

Central fringe :

At the point of contact of the lens and the glass plate $d = 0$. Consequently, the condition (9.16-3) for destructive interference will be satisfied with $m = 0$. This indicates that *the central fringe is dark* and is situated at the point of contact of the lens and the glass plate. It appears as a dark spot.

Newton's rings with white light :

With a white light the central spot will be black and it will be surrounded by a few (8 -10) coloured rings and beyond this there will be general illumination due to overlapping of different coloured rings.

Determination of wavelength :

Newton's rings can be used for the measurement of the wavelength of monochromatic light. The diameters of mth Newton's ring for an air film ($n = 1$) are given from Eqs. (9.16-7) and (9.16-8) as

$$D_m^2 = 2(2m+1)\lambda R \text{ [Bright ring]}$$

$$D_m^2 = 4m\lambda R \text{ [Dark ring]} \qquad \ldots(9.16\text{-}10)$$

Thus if D_m and D_{m+p} are respectively the diameters of the mth and $(m+p)$th rings (bright or dark) then the wavelength λ is given by

$$\lambda = \frac{D_{m+p}^2 - D_m^2}{4pR} \qquad \ldots(9.16\text{-}11)$$

So by measuring the diameters D_m, D_{m+p} of Newton's rings and counting the number p we can determine λ.

Experiment :

Experimental arrangement to measure the wavelength of monochromatic light by Newton's ring apparatus is shown in Fig. 9.16-3. Light from the monochromatic source S is made parallel by putting the source at the focal plane of the convex lens C. These parallel rays after being reflected by a glass plate, P, kept inclined to the horizontal by an angle of 45°, fall normally on the air film enclosed between the plano-convex lens L and the glass plate G. Newton's rings are viewed vertically by a low power microscope M placed above the glass plate P.

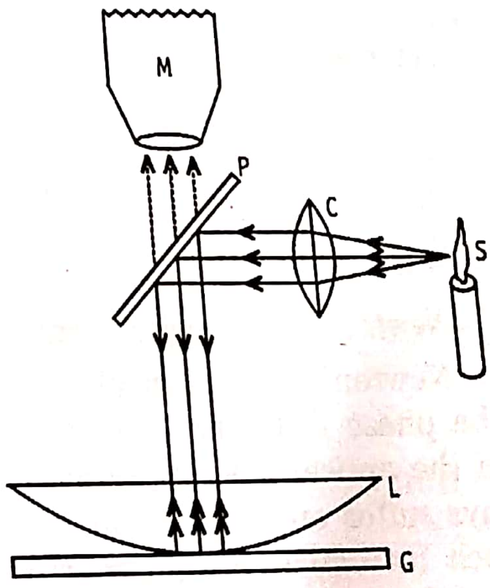


Fig. 9.16-3

Any bright ring is selected and one of the cross-wires of the microscope is made tangential to one edge of this bright ring and the reading of the vernier attached to the microscope is noted. The microscope is then laterally shifted until the same wire is tangential to the other extremity of the same bright ring. Again t reading of the vernier attached to the microscope is noted. Th difference of these two vernier readings gives the diameter D_m (se of this ring. Then again the diameter of another bright ring, separate from the former bright ring by a known number p of dark rings, similarly measured. Let this diameter be D_{m+p}.

The radius of curvature R of the lower surface of the plano-conve lens L is measured by a spherometer or by Boy's method. Th wavelength λ of monochromatic light is then obtained from the relatio (9.16-11). Usually the radius R is first determined by using the formul (9.16-11) and a light of known wavelength, say sodium light. Then wit this determined value of R, the wavelength (λ) of any unknown ligh is determined by the relation (9.16-11).

Determination of refractive index of a liquid :

Newton's rings can also be used to measure the refractive index (n) of a liquid. At first the diameters of mth and $(m + p)$th bright or dark rings are measured with air film. Then the diameters of these rings are measured again by forming a liquid film between the lens and the glass plate. Introduction of the liquid decreases the diameters of the rings.

For air film,

$$\left(D_{m+p}^2 - D_m^2\right)_{air} = 4p\lambda R$$

For liquid film,

$$\left(D_{m+p}^2 - D_m^2\right)_{liq} = \frac{4p\lambda R}{n}$$

Hence,

$$n = \frac{\left(D_{m+p}^2 - D_m^2\right)_{air}}{\left(D_{m+p}^2 - D_m^2\right)_{liq}} \qquad \ldots(9.16\text{-}12)$$

Newton's rings with transmitted light :

Newton's rings can also be observed with transmitted light. In this the phase difference between the transmitted waves will be only due to the optical path difference $2.n.d$. This is so because the transmitted rays suffer only refraction or refraction plus even number of reflections each introducing a phase change of π (See, Fig. 9.16-1). Consequently for normal incidence,

$$2.n.d = 2m.\lambda/2 \text{ for maximum} \qquad \ldots(9.16\text{-}13)$$

$$2.n.d = (2m+1)\lambda/2 \text{ for minimum} \qquad \ldots(9.16\text{-}14)$$

For central fringe $m = 0$ and this is satisfied by the Eq. (9.16-13) with $d = 0$. Thus the central spot is bright and is situated at the point of contact of the lens and the glass plate.

Comparing the conditions (9.16-13) and (9.16-14) with the corresponding conditions (9.16-2) and (9.16-3) with reflected light we can conclude that the rings observed with transmitted light are exactly complementary to those seen with reflected light. However, the dark rings with transmitted light are not completely dark and hence the transmitted pattern is not so distinct as observed with reflected light. For this Newton's are usually observed by reflected light.

9.17 NEWTON'S RINGS UNDER DIFFERENT CONFIGURATIONS :

(a) Lens separated from the plate by some distance :

Let a ray PQ of a parallel beam of light of wavelength λ be incident almost normally on the upper surface of a wedge shaped film enclosed between the convex surface of a lens L_1 and a glass plate G. If x be the distance between the upper plane face of the plate and the tangent plane drawn at the lowest point of the convex surface of the lens L_1 then thickness of the air film at Q is given by

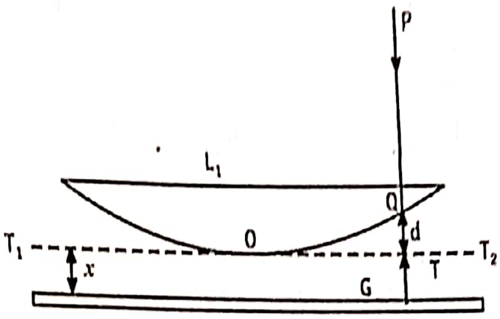


Fig. 9.17-1

$$d' = d + x$$

If the point Q is situated on the mth order bright ring then [See Eq. (9.16-2)]

$$2nd' = 2n(d+x) = (2m+1)\lambda/2 \quad \text{...(9.17-1)}$$

From Eq. (9.16-4), $\quad d = r_m^2/2R$

Therefore from Eq. (9.17-1),

$$2nx + \frac{n.r_m^2}{R} = (2m+1)\lambda/2 \quad \text{...(9.17-2)}$$

Similarly the radius r_{m+p} of the $(m+p)$th order bright ring is given by

$$2nx + \frac{n.r_{m+p}^2}{R} = (2m+2p+1)\lambda/2 \quad \text{...(9.17-3)}$$

Taking the difference of Eqs. (9.17-2) and (9.17-3) we get

$$\lambda = \frac{\left(r_{m+p}^2 - r_m^2\right)n}{p.R}$$

In terms of diameters,

$$\lambda = \frac{n\left(D_{m+p}^2 - D_m^2\right)}{4p.R} \qquad \ldots(9.17\text{-}4)$$

This equation may be employed to find λ experimentally.

If the lens is slowly moved upward the rings will be found to converge at the centre. By moving the lens up through a known distance and counting the number of rings that have converged to the centre it is also possible to determine λ.

(b) Lens in contact with a concave surface :

Let a ray PQ of a parallel beam of light of wavelength λ be incident almost normally at Q on the upper face of the air film enclosed between the lower convex surface of a plano-convex lens L_1 and the concave surface of plano-concave lens L_2. The two surfaces are in contact at the point O. A common tangent T_1OT_2 is drawn to the two surfaces at the point of contact O (Fig. 9.17-2).

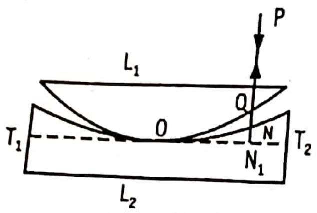


Fig. 9.17-2

If d be the thickness of the air film at Q then $d = QN_1 - NN_1$. From the geometry of Fig. 9.17-2, we get, $QN_1 = ON_1^2/2R_1$ and $NN_1 = ON_1^2/2R_2$; where R_1 and R_2 are respectively the radii of curvature of the convex surface of L_1 and the concave surface of L_2. Hence the value of d is,

$$d = QN_1 - NN_1 = \frac{ON_1^2}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{r_m^2}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \ldots(9.17\text{-}5)$$

If Q be situated on the mth bright ring then $ON_1 = r_m$ would be the radius of the mth bright ring. Again if Q be on the bright ring of mth order then,

$$2.n.d = (2m+1)\lambda/2$$

or,

$$n.r_m^2\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (2m+1)\lambda/2 \qquad \ldots(9.17\text{-}6)$$

Similarly the radius r_{m+p} of $(m+p)$th bright ring is given by

$$n.r_{m+p}^2\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (2m+2p+1)\lambda/2 \qquad \ldots(9.17\text{-}7)$$

Taking the difference of Eqs. (9.17-6) and (9.17-7) we get

$$\lambda = \frac{n\left(r_{m+p}^2 - r_m^2\right)}{p}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

In terms of diameters,

$$\lambda = \frac{n\left(D_{m+p}^2 - D_m^2\right)}{4p}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \text{...(9.17-8)}$$

This relation may be employed to determine λ. Note that the above relation is also true when dark rings are selected.

(c) Lens in contact with a convex surface :

In Fig. 9.17-3 the convex surface (of radius R_1) of the plano-convex lens L_1 is in contact with another convex surface (of radius R_2) of the plano-convex lens L_2. The point of contact of these two convex surfaces is at O. A common tangent TOT_1 is drawn to the two surfaces at O.

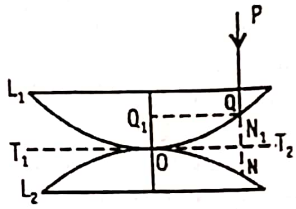


Fig. 9.17-3

A ray PQ of a parallel beam of light of wavelength λ is incident almost normally at Q on the air film enclosed between the two convex surfaces. If Q be situated on the mth bright ring then the radius r_m of this mth bright ring would be, $r_m = Q_1Q = ON_1$. The thickness of the air film at Q is, $d = QN_1 + N_1N$. From the geometry of the Fig. 9.17-3,

we get, $$QN_1 = \frac{r_m^2}{2R_1}$$

and $$N_1N = \frac{r_m^2}{2R_2}$$

Hence, $$d = \frac{r_m^2}{2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \qquad \text{...(9.17-9)}$$

Again if Q be the position of the bright ring of mth order then,

$$2nd = (2m+1)\lambda/2$$

or, $$n.r_m^2\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = (2m+1)\lambda/2 \qquad \text{...(9.17-10)}$$

Similarly the radius r_{m+p} of $(m+p)$th bright ring is given by,

$$n.r_{m+p}^2\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = (2m+2p+1)\lambda/2 \qquad \text{...(9.17-11)}$$

Taking the difference of the Eqs. (9.17-10) and (9.17-11) we get,

$$\lambda = \frac{n\left(r_{m+p}^2 - r_m^2\right)}{p}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

In terms of diameters of the rings,

$$\lambda = \frac{n\left(D_{m+p}^2 - D_m^2\right)}{p}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \qquad \ldots(9.17.$$

9.18 WIENER'S EXPERIMENT :

In 1890 Wiener demonstrated that when light is reflected from a surface the incident and reflected waves combine to form a stationary wave pattern. In his experiment Wiener took an extremely thin film of photographic emulsion kept inclined at a small angle with a perfectly reflecting surface. When the