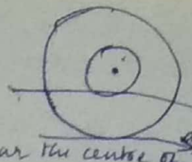
NUCLEAR PHYSICS



The discovery of atomic nucleus — was made by Rutherford in 1911 through his α-particle scattering experiment, when positively charged α particles were scattered by thin metallic foils made of gold $(Z = 79)$ or silver $(Z = 47)$. He concluded that the scattering was due to electrostatic interaction between the positive charge of α particle and a concentrated positive charge region within the atom. This concentrated +ve charge region inside at the centre of atom is named as nucleus. It is designated by the symbol $^{A}_{Z}X_{N}$

Angle of scattering increases with increasing effective charge of the atom.

- The radius of the nucleus is of the order of 10^{-14} to 10^{-15} mt. while the radius of the atom is about 10^{-10} m.
- Nucleus is composed of two types of nucleons — neutrons, protons where proton carries positive charge and neutrons are uncharged. and they have roughly the same mass.
- Negatively charged electrons move around the nucleus.
- The negative charge of an electron is equal to the magnitude of the charge of proton.
- The mass of the electron is determined to be about 1/1836 times that of proton. So more than 99.99% of the mass of the atom is concentrated within the small volume of the nucleus.
- The number of protons in the nucleus are denoted by Z. So the charge of nucleus is Ze. and $Z \propto A$
- The number of neutrons in the nucleus is denoted by N,
- So total number of nucleons in the nucleus $= N + Z = A$, atomic mass number, and mass of nucleus $= Zm_p + Nm_n$.

$1\ amu = 1.66054\times10^{-27} Kg = 931.49\ MeV/c^2$.

- The neutron and proton have nearly same mass, neutron mass being 1.008665 amu $= 1.675\times10^{-27}$ kg $= 939.566\ MeV/c^2$ $= 1838.679\ m_e$ and proton mass being 1.007276 amu $= 1.672\times10^{-27}$ kg $= 938.272\ MeV/c^2 = 1836.149\ m_e$. Where a.m.u. is defined as the $\frac{1}{12}$th mass of carbon-12 nuclear mass or 1.660566×10^{-27} kg. 1 amu = 931.49 MeV/c^2.
$m_e = 0.511\ MeV/c^2$.

- Nuclear force; that holds the nucleons together is very short range force which decreases rapidly with separation of particles and becomes insignificant for distances greater than a fermi ($10^{-15}m = 1$ fermi)

- Nuclear force is independent to the electric charge. i.e. proton-neutron (p−n), and neutron-neutrons (n−n), proton-proton p−p. forces are equal.

The nuclear radius $R = R_0 A^{1/3}$, infact nuclear radius is the radius of the nuclear mass distribution where R_0 (Nuclear radius parameter) varies slightly from one nucleus to another but is roughly constant for $A > 20$.

Experimental evidence shows that $R_0 = (1.2 \pm 0.1)\times10^{-15}$
$= 1.3\times10^{-15}$ mt.

The variation in R_0 is from 1.2 to 1.5 fermi (1 fermi $= 10^{-15}$ m) for different nuclei. i.e. $R_0 = 1.2\times10^{-15}$ m $= 1.2$ fermi

Assuming spherical symmetry, the nuclear volume V is

$$V = \frac{4}{3}\pi R^3$$

but $R = R_0 A^{1/3}$. so, $V = \frac{4}{3}\pi R_0^3 A$; R_0 is known as nuclear radius parameter.

Mass of one proton $= 1.67\times10^{-27}$ kg.

$\therefore$ Nuclear mass $= A \times 1.67\times10^{-27}$ kg.

So Nuclear density $= \dfrac{A\times1.67\times10^{-27}}{\frac{4}{3}\pi R_0^3 \times A}$ kg $= \dfrac{M}{V} = \rho$

$= \dfrac{1.67\times3\times10^{-27}}{4\times3.14\times R_0^3}$ kg/m^3 $\approx 2\times10^{17}$ kg/m^3.

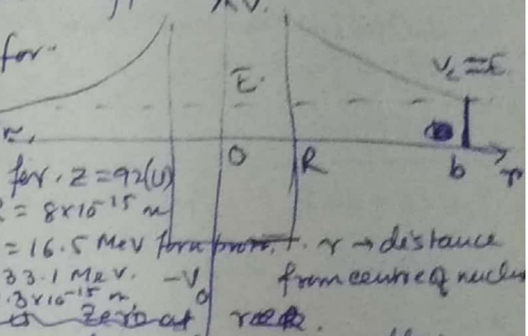
nuclear particle density (Number of nucleons / m^3).

$$= \frac{\text{Nuclear mass density}}{\text{nucleon mass}} = \frac{2\times10^{17}\ kg/m^3}{1.67\times10^{-27}\ kg}$$

$\approx 10^{44}$ nuclons/mt^3.

So nucleus is a very tightly bound system of particles with a large potential energy which offsets the K.E. of the nucleons.

P.E diagram for proton/α particle:
$r > R \rightarrow$ repulsive, $r < R \rightarrow$ attractive.



for $Z = 92$(U)
$R = 8\times10^{-15}$ m
$V_R = 16.5$ MeV for a proton.
$V_R = 33.1$ MeV.
$R_0 = 1.3\times10^{-15}$ m
$r \rightarrow$ distance from centre of nucleus
~~nuclear force is zero at~~ r=R.

$$\rho_{atom} = \frac{\rho_{nuclei}}{10^{12}} \approx 2\times10^{5}\ kg/m^3$$

assume, electrostatic force is not effective inside nucleus and at $r=R$ nuclear force is zero,

• The 'electrical radius' of the nucleus and 'nuclear matter radius' are approximately same, i.e. the electrical charge distribution is same as nuclear mass distribution within the nucleus.

$V_C = \frac{ZZ'e^2}{4\pi\epsilon_0 r}$ is coulomb potential barrier for an incident particle of charge $Z'e$ at $r > R$

and at $r = R$ this barrier height is, $V_R = \frac{ZZ'e^2}{4\pi\epsilon_0 R}$

There are two ways to find out the size of nucleus. ⇒

1) Electrical Method —
A) Mesonic X ray
B) Electron scattering
C). Coulomb energies of mirror nuclei
D).

2). Nuclear Method :—
1) Neutron scattering
2) α decay.
3). α- particle scattering
4). Isotopic shift in line spectra.

• Except for the lightest nuclei, the charge distribution is also found to be approximately spherical with the relation

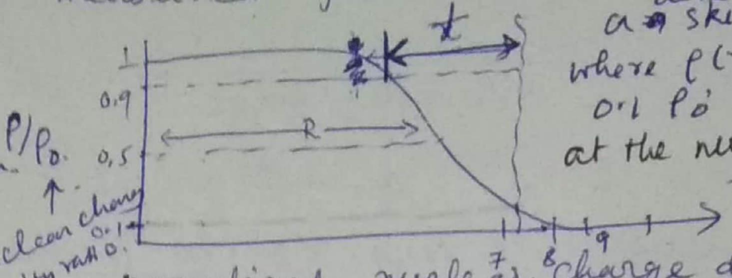
$$\rho(r) = \frac{\rho_0}{1+e^{(r-R_{1/2})/a}}$$

This is known as Fermi distribution. $R_{1/2}$ and a are adjusted to best fit of experimental data.

where ρ_0 = central nuclear charge density, $R_{1/2}$ is the radius where the charge density drops to half its central density, and $t = 4.4a$. The

where $a = 0.5\times10^{-15}$ mt.

Surface thickness measured from 90% to 10% of the central density.



$\Rightarrow$ The mean squared radius of nuclear charge distribution.

ρ/ρ_0 ↑ nuclear charge density ratio.

$$\langle r^2 \rangle = \frac{\int_0^\infty r^2 \, 4\pi r^2 \rho(r)\, dr}{\int_0^\infty 4\pi r^2 \rho(r)\, dr}$$

a determines skin thickness of nucleus where $\rho(r)$ falls to $0.9\rho_0$ to $0.1\rho_0$ i.e $t = 4.4a$ at the nuclear surface. radial dist.

normalized nuclear charge density

a = same for all nuclei = 0.5×10

for $A = 150$

for uniformly charged sphere $\rho = $ const and $\rho = 0$ for $r > R$ $\therefore$ $\langle r^2 \rangle = \frac{\int_0^R r^4 dr}{\int_0^R r^2 dr} = \frac{3}{5} R^2$. $\therefore R^2 = \frac{5}{3} \langle r^2 \rangle$. $= 0.5$

Isotope:-

Elements with same atomic number but different mass number, e.g.:- $_1H^1$, $_1H^2$, $_1H^3$ of Hydrogen

$_{17}Cl^{37}$ / $_{17}Cl^{35}$

They are chemically same but physically different.

Isobar:-

Isobars are chemically different but physically same.
These are the elements with same atomic mass but different atomic number.
Since their number of electrons are different, so their chemical properties are different.

e.g, 1) $_{26}Fe^{58}$, $_{27}Ni^{58}$ (2) $_8O^{18}$, $_9F^{18}$

3) $^{23}_{10}Ne$, $^{23}_{11}Na$, $^{23}_{12}Mg$

Isotones:-

Elements with same number of neutrons.

e.g. 1) $_{17}Cl^{37}$, $_{19}K^{39}$ both have 20 neutrons.

2) $_6C^{14}$, $_7N^{15}$ both have 8 neutrons.

3) $^{13}_5B$, $^{14}_6C$, $^{15}_7N$, $^{16}_8O$

Neutron, proton, electron, positron, μ meson, neutrino → F.D (Fermions)
photons, α particles, π meson → B.E (Bosons).

BINDING ENERGY:-

Particles which constitute the stable nucleus are held together by strong attractive forces and in order to separate them apart, work must be done, i.e. energy must be supplied to the nucleus to separate it into individual constituents.

From Einstein's mass energy relation we know $E = mc^2$.
~~From~~ we can expect the total mass of the nucleus to be less than the sum of the constituents.

The Binding Energy of the nucleus is defined as the difference between the energy of the constituent particles and whole nucleus.

i.e. $B = [ZM_p + NM_n - {}_Z M^A] c^2 = \Delta M c^2$ where ${}_Z M^A$ = mass of nucleus.

where M_p = mass of proton; M_n = mass of neutron.

Z = Number of protons, $N = A - Z$ = Number of neutrons.

${}_Z M^A$ = measured mass of ~~neutral atom~~ atom $= M(Z, A)$

Now $M_p = M_H - M_e$ so $ZM_p = Z(M_H - M_e)$. so ${}_Z M^A$ = mass of nucleus $= M_a - ZM_e$, M_a = Mass of atom.

the binding energy can also be written as;

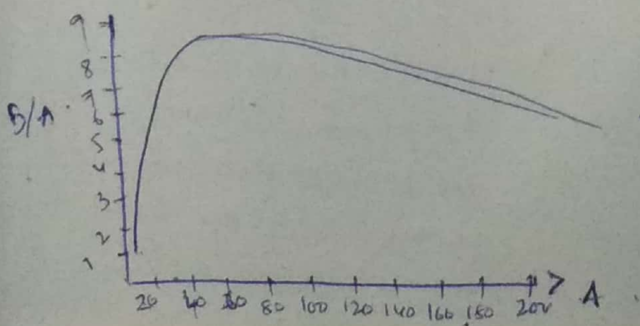
$$B = [ZM_H + NM_n - M_a] c^2.$$

M_H = mass of neutral hydrogen atom.

since there are A nucleon in the nucleus the binding energy per nucleon is

$$\frac{B}{A} = \frac{c^2}{A}[ZM_H + NM_N - M(Z, A)]$$

If Binding energy > 0 → Stable
< 0 → unstable, it will disintegrate spontaneously



the curve is almost const for $A = 30$ to $A = 100$ and decreases for small and large values of A.

The decrease of B/A for large A is due to Coulomb repulsion between the protons which make the nuclei less stable.

The decrease of $\frac{B}{A}$ for light nuclei, → Individual nucleons are attracted by only a few other nucleons and hence their distances of separation are larger which again reduces the stability.

It is also found that particles at the surface are less strongly bound than those in the interior.

The smaller the nucleus, the larger is the percentage of constituents at nuclear surface. This effect reduces $\frac{B}{A}$ for low A.

• ${}_1H^3$, ${}_3Li^6$, ${}_5Bi^{10}$, ${}_7N^{14}$ → examples of odd-nuclei which are most-stab

Packing Fraction :-

$$= \; f = \frac{\text{Atomic mass} - \text{Mass number}}{\text{Mass Number}} = \frac{M - A}{A} = \frac{\Delta M}{A}$$

where (M − A) is known as mass defect. So packing fraction is the ratio of mass defect to mass number which is used to measure the comparative stability of the atom.

or, $M = A(1+f)$

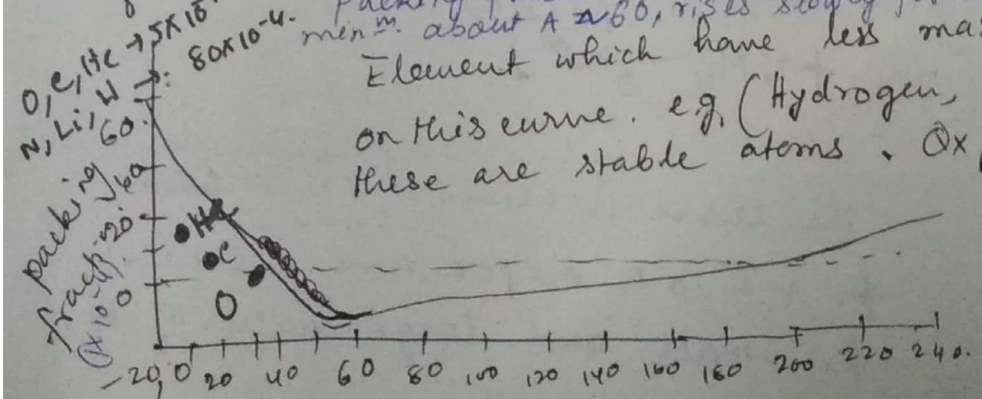
for light atom $A < 20$, and for heave atom $A > 180$, Atomic mass > A.

(+ve) packing fraction describes a tendency towards instability

(−ve) " " means isotopic / atomic mass is less than mass number. This difference is due to transformat of mass into energy in the formation of nucleus.

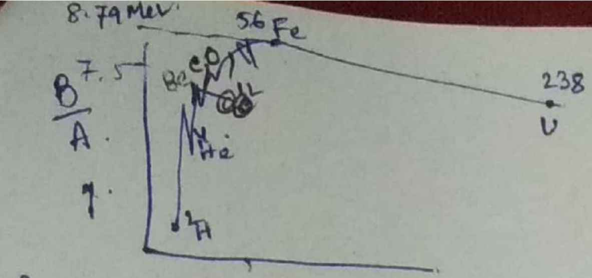
Packing fraction becomes (−ve) for A greater than about 20, attains a minm. about A ≈ 60, rises slowly for higher A and (+ve) again for A > 180

Element which have less mass number does not fall on this curve. e.g. (Hydrogen, helium, Carbon) Therefore these are stable atoms. Oxygen is highly stable → cuts at zero



packing fraction beyond mass number 200 becomes positive and increases with increase in mass number and instability increase.

for higher A, Stability occurs at even-even pair for low A, stability N = Z,



1) The curve rises steeply initially and attains a value of ~8 MeV/nucleon for A~20, then gradually reaches maximum of 8.79 MeV at A = 56. This number A = 56 corresponds to Iron Fe^{56}_{26}.

$\frac{B}{A} = f_B$ = binding fraction which represents the relative strength of the binding of nucleons.

3) for A > 180
The curve then drops very slowly to somewhere around 7.6 MeV at the highest known mass number.

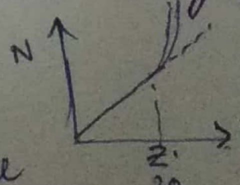
- So it can be inferred that nuclei of intermediate mass is most stable since high amount of energy is required to separate these nucleons. For 2) 20 < A < 180 curve varies slightly, mean value of 8.5 MeV, peaks are observed for even-even nuclei ^4He, ^8Be, ^{12}C, ^{16}O etc. for which A = 4n. Similar peak but less are observed for magic number values Z or N = 20, 28, 50, 82...
- For heaviest nuclei f_B is about 7.5 MeV/nucleon.
- Appearance of the peak shows greater stability.

Nuclear stability:

The stability of nuclei are classified according to even and odd number of protons and neutrons.

PROTONS	Neutrons	Stable Nuclei
EVEN	EVEN	164
EVEN	ODD	54
ODD	EVEN	50
ODD	ODD	4
		272 total

- For stable nuclei, nature prefers even number of protons and an even number of neutrons. Nuclei with even A have frequently two and sometimes three stable isobars.
- Odd - odd combination of proton and neutron in the nucleus are very less
- For elements with large number of proton in the nucleus, the coulomb electrostatic force of repulsion becomes significant and the number of neutrons must be greater to compensate this repulsion effect.



highest value of $\frac{N}{Z} = 1.6$ for heavy nuclei

- Nuclei with odd A have only one stable isobar
- For light nuclei upto $_{20}Ca^{40}$ N = Z (see plot) ⇒ p-p and n-n forces are equal known as charge symmetry.

for heaviest stable nucleus $^{208}_{82}Pb_{126}$ → N − Z = 126 − 82 = 44

Magic number of nuclei are 2, 8, 20, 50, 82 and 126. ∴ Thus Tin ($Z = 50$) has 10 stable isotopes whereas Indium ($Z = 49$) and antimony ($Z = 51$) have only 2 stable isotopes. This numbers are such that they are arranged into complete shells within the nucleus.

Non-existence of electrons in the nucleus.

First assume that, electron resides in the nucleus. then apply Heisenberg uncertainty principle.

The radius of nucleus is nearly 10^{-14} m.

So uncertainty in the position of electron is $\Delta x = 10^{-14}$ m

Now $\Delta x \Delta p = \hbar = \frac{h}{2\pi}$

$$\therefore \Delta p = \frac{h}{2\pi(\Delta x)} = \frac{6.6 \times 10^{-34}}{2 \times 3.14 \times 10^{-14}}$$

or, $\Delta p = 1.05 \times 10^{-20}$ Kg m/sec.

An electron having this much high momentum must have a velocity comparable to the velocity of light. So E will be,

$$E = \sqrt{m_0^2 c^4 + p^2 c^2}$$

$$E = \sqrt{(9.1 \times 10^{-31})^2 \times (3 \times 10^8)^4 + (1.05 \times 10^{-20})^2 \times (3 \times 10^8)^2}$$

$$E = \sqrt{9.9267 \times 10^{-24}}$$

$$\therefore E = 3.15 \times 10^{-12} \text{ J} = \frac{3.15 \times 10^{-12}}{1.6 \times 10^{-19}} \text{ eV}.$$

$$= 19.6 \times 10^{6} \text{ eV} = 19.6 \text{ MeV}.$$

So, if an electron exists in the nucleus its energy will be as high as 19.6 MeV.

But experimental results show that no electron or particle in the atom possess ~~energy~~ energy greater than 4 MeV.

So, it is confirmed that electrons do not reside in nucleus.

the highest mean binding energy per nuclear particle never exceeds 8.9 MeV.

∴ [from e-p hypothesis $^{14}_{7}N$ would have 14 (A) no of protons and 7 (A−Z) no of electrons so that 21 no of nucleons in the nucleus]

Nuclear Models.

Constitution of the nucleus:

1) At first it was assumed that as the atomic mass of $^1H_1$ of hydrogen is nearly unity and it is nothing but a proton so a nucleus of mass number A can be made up of A number of Hydrogen atom / protons. Then the total charge of atom will be $+Ae$. but actually it is less than $+Ae$ and it is $+Ze$. So this assumption was wrong.

2) a) Then it was assumed apart from A protons, there is $(A-Z)$ electrons in the nucleus to have the total charge of nucleus $+Ze$. and as the mass of the electron is much smaller than that of proton, the total mass of the nucleus would be nearly A. But this proton-electron hypothesis also proved to be wrong. as discussed earlier (from Δp consideration)

b) again from consideration of angular momentum of nuclei the proton-electron hypothesis would pose serious difficulties. Electron and proton both are spin $\frac{1}{2}$ particles. Their total number in the nucleus $= A+(A-Z) = 2A-Z$. If this is even, then total spin (I) should be integral " " " odd " " " (I) " " half integral.

because $I = L+S$. where L is orbital angular momentum which is integral and S is resultant intrinsic spin angular momentum of all the particles in the nucleus which can be integral or half integral, depending on the total number of particles in nucleus is even or odd.

e.g. for $^{14}N_7$, $Z=7$ $\therefore$ $2A-Z = 28-7 = 21$ which is odd.

So $^{14}N_7$ would have $\frac{1}{2}$ integral spin but experiment gives $I=1$ which is contrary to proton-electron hypothesis.

c). The intrinsic magnetic moment of the electron is about 1000 times larger than that of nucleons. If the nucleus contains electrons, then its magnetic moment should be of the order of electronic magnetic moment, But the measured value is much smaller.

d) All elementary particles in nature ~~obey~~ are fermions or bosons on the basis of the symmetry property of their wave functions.

particles → half integral intrinsic spin → antisym wavefn → F.D. stat
(total no of electron + proton odd) → fermions.

→ integral (0, 1, 2) spin → sym wave fn → B.E. stat → Bosons
(total no of electron and proton even).

A composite isolated system containing N fermions will obey either F-D stat or BE stat depending on whether total number of protons and electrons are odd or even.

e.g. for $^{14}N_7$ nucleus, there would be 21 protons and electrons (for proton-electron hypothesis) which is odd and should obey F.D stat but experimental evidence shows it obeys B.E stats.

So all the above evidences are against the proton-electron hypothesis

- Then in 1932 Rutherford's student James Chadwick observed the emission of neutral particle from the nucleus while performing an experiment on artificial transmutation of elements which he named neutrons which is fermion because of spin $\frac{1}{2}$.

- After this discovery W. Heisenberg (1932) proposed that nuclei are made up of protons and neutrons.

The observed spin, magnetic moment, ~~etc~~ are well explained on the basis of proton-neutron hypothesis.

e.g. $^{14}N_7$ → total number of particles in nucleus is 14 (even) so spin I will be integral and obeys B.E stats which agrees with experiment.

Nature of nuclear force :-

- The protons and neutrons are very strongly bound within the nucleus. and it is different from gravitational or electromagnetic forces.

The gravitational force is too weak to account for nuclear binding. The gravitational interaction between two nucleons. The P.E of gravitational interaction within the nucleus at a distance of 2×10^{-15} m is

$$V_g = \frac{G(1.65\times10^{-27})^2}{(2\times10^{-15})} = 6.672\times10^{-11}\times 1.3778\times10^{-39}$$

$$= 5.75\times10^{-32}\ MeV.$$

which is smaller than the binding energy per nucleon. which is of the order of a few million electron volt

- Again for em force two protons are repelling but inside the nucleus it is attracting.

- The nuclear force is attractive upto a certain distance of the order of 2 fm. This distance is range of the force.

- According to H. Yukawa in 1935, a pion or π meson is exchanged between two nucleons when they are at a distance less than about 2 fm. (exchange force).