

Study Material

Subject: Mathematics

Semester: 2nd

Name of Teacher: Prabir Rudra

Topic: Permutation Groups (Assignment) (CC-4)

Advice from faculty

The students of 2nd semester (Mathematics honours) are advised to attempt the enclosed assignment on Permutation groups. We have already conducted an extensive doubt clearing session on this topic via video conferencing on 30.04.2020. In case you have further queries while doing this assignment you can consult me over mail, WhatsApp or Google Classroom. You may submit the assignment in Google classroom or over mail (asutoshcollegemath@gmail.com) by 17.05.2020.

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Date: 09.05.2020

PERMUTATION GROUPS

Date - 09/05/2020

Assignment

1) Consider the following permutations in S_6 .

$$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 4 & 5 & 6 & 3 \end{pmatrix}, \quad \beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 2 & 4 & 1 & 6 & 5 \end{pmatrix} \text{ and}$$

$$\gamma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 3 & 2 & 5 & 1 & 6 \end{pmatrix}$$

Then compute each of the following:

(i) $\alpha\beta$ (ii) $\beta\alpha$ (iii) $\alpha\gamma^2$ (iv) γ^{-1} (v) $\gamma\alpha\gamma^{-1}$

2) Let $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 6 & 4 & 7 & 5 & 2 & 3 & 1 \end{pmatrix}$, $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 4 & 6 & 7 & 3 & 5 & 2 \end{pmatrix}$

be elements of S_7

- (i) Write α as a product of disjoint cycles.
- (ii) Write β as a product of 2-cycles.
- (iii) Is β an even permutation?
- (iv) Is α^{-1} an even permutation?

3) Let $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 2 & 4 \end{pmatrix}$. Find the smallest positive integer k such that $\alpha^k = e$ in S_4 , where e is the identity permutation in S_4 .

4) Compute each of the following:

- (i) $(1423)(34)(1324)$ in S_4
- (ii) $(1254)(243)(12)$ in S_5

5) Show that the number of even permutations in S_n ($n \geq 2$) is the same as that of the odd permutations.

- 6) If $\beta = (1\ 2\ 3)(1\ 4\ 5)$, write β^{99} in cycle notation.
- 7) Let $\beta = (1\ 3\ 5\ 7\ 9\ 8\ 6)(2\ 4\ 10)$ in S_{10} . What is the smallest positive integer n for which $\beta^n = \beta^{-5}$?
- 8) ~~1~~ Determine whether the following permutations are even or odd.
- (a) $(1\ 2\ 3\ 4)$ (b) $(2\ 3\ 4)$ (c) $(1\ 2\ 3\ 8)(3\ 5\ 8)(1\ 3)$
- (d) $(1\ 2\ 3\ 4)(1\ 2\ 5\ 3)$ (e) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 1 & 3 & 5 & 4 & 7 & 6 & 8 \end{pmatrix}$
- 9) In S_6 , let $f = (1\ 2\ 3)$ and $\sigma = (4\ 5\ 6)$. Find a permutation x in S_6 such that $x f x^{-1} = \sigma$.

Multiple choice Questions (Give proper justification)

- 10) (i) The order of the permutation $(1\ 2\ 3\ 4)(5\ 6) \in S_6$ is
(a) 6 (b) 2 (c) 3 (d) 4
- (ii) The number of elements of order 3 in A_4 is
(a) 1 (b) 2 (c) 7 (d) 8
- (iii) The number of elements of order 6 in S_4 is
(a) 1 (b) 2 (c) 6 (d) 0
- (iv) The number of elements of order 2 in A_4 is
(a) 1 (b) 2 (c) 3 (d) 6
- (v) The number of elements in an alternating group A_n is
(a) $n!$ (b) $\frac{n!}{4}$ (c) 2^n (d) $\frac{n!}{2}$