Subject Name: Statistics

Semester/Year: IInd, Ist year

Name of the Teacher: SAIKH RUPA RAHILA

Name of Topic: Harmonic Mean

What is Harmonic Mean?

Harmonic mean is a type of average that is calculated by dividing the number of values in the data series by the sum of reciprocals (1/x_i) of each value in the data series. A harmonic mean is one of the three Pythagorean means (the other two are arithmetic mean and geometric mean). The harmonic mean always shows the lowest value among the Pythagorean means.

Formula for Harmonic Mean

The general formula for calculating a harmonic mean is:

Harmonic mean = n / (∑1/x_i)

Where:

- n – the number of the values in a dataset
- x_i – the point in a dataset

Problem: During one month the total number of kilometres driven by each truck is given below. Find the geometric mean.

Truck Number	1	2	3	4
Km. driven	40	50	60	75

x	$1/x$
40	0.02500
50	0.02000
60	0.01677
75	0.01333
	0.07500

$$H.M = \frac{N}{\sum 1/x}$$

$$H.M = \frac{4}{0.07500}$$

$$H.M = 53.33\ Km.$$

Harmonic Mean of grouped Data

Harmonic mean of grouped data can be calculated with the help of following formula:

$$H.M = \frac{N}{\sum f/x}$$

Here,

N is the sum of all frequencies

f is the frequency corresponding to each observation x while ?f/x represents the sum of reciprocal of grouped observations.

Problem: Following is the frequency distribution of the marks (out of 40) obtained by the students of a certain college in statistics. Calculate harmonic mean.

Class Interval	11 -- 15	16 -- 20	21 -- 25	26 -- 30	31 -- 35
Frequencies	15	20	60	150	15

Class Interval	x	f	f/x
11 – 15	13	15	1.153846154
16 – 20	18	20	1.111111111
21 – 25	23	60	2.608695652
26 – 30	28	150	5.357142857
31 – 35	33	15	0.454545455
		260	10.68534

$x = Midpoint$

$$x = \frac{L.C.L + U.C.L}{2}$$

$$e.g\ x = \frac{11 + 15}{2} = 13$$

$$H.M = \frac{N}{\sum f/x}$$

$$H.M = \frac{260}{10.68534} = 24.3$$

Merits of H.M:

- It is rigidly defined
- It is based on all the observations of the series
- It is suitable in case of series having wide dispersion
- It is suitable for further mathematical treatment
- It gives less weight to large items and more weight to small items

Limitations of H.M:

- It is difficult to calculate and is not understandable
- All the values must be available for computation
- It is not popular due to its complex calculation.
- It is usually a value which does not exist in series

When to use?

Harmonic mean is used to calculate the average value when the values are expressed as value/unit. Since the speed is expressed as km/hour, harmonic mean is used for the calculation of average speed.

Relationship among the averages:

In any distribution when the original items are different the A.M., G.M. and H.M would also differ and will be in the following order:

A.M. ≥ G.M ≥ H.M