ATMOSPHERIC DISPERSION MODELING:

(1)

(1) BOX MODEL :

At a first approximation, the atmosphere over an urban area can be modelled as a well-mixed box with horizontal dimension 'Δx' & 'Δy' & height 'H' as shown in the Fig. 1. This type of model is known as 'Box model'.

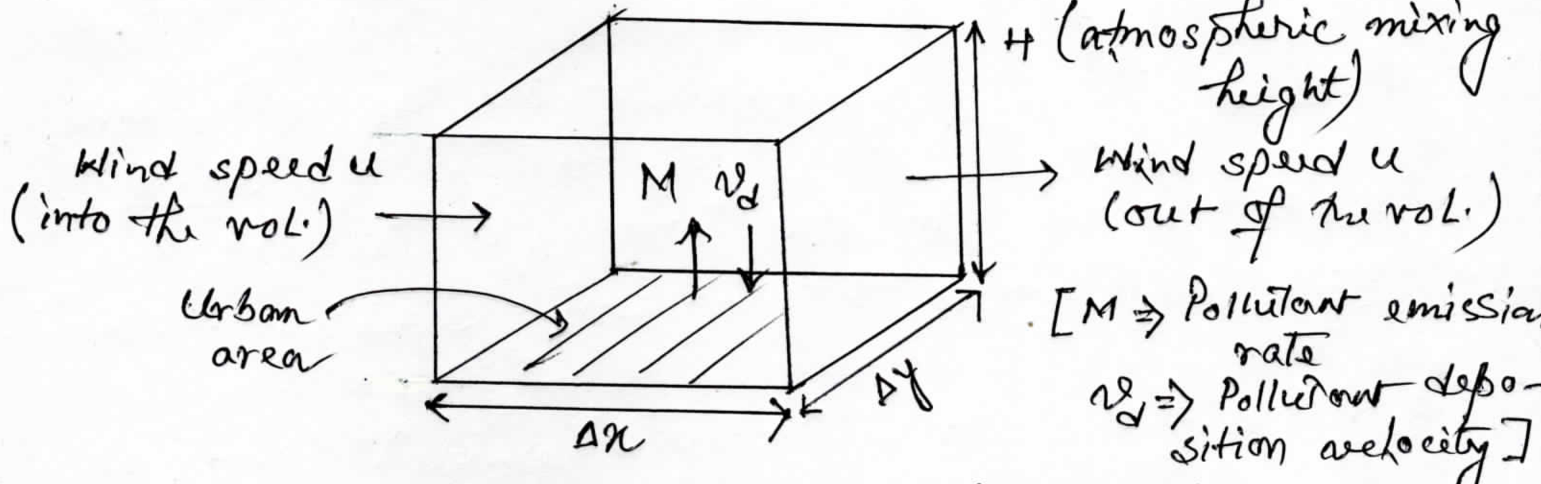

Fig. 1. Urban area in a box model.

Let us also assume that, the wind speed is 'u' & the wind direction is parallel to the dimension of the city.

Troposphere is the lowest layer of the atmosphere & contains most of the atmospheric gases. It is extended up to 10 km. but it is not well mixed in urban areas. Typically, the mixing height 'H' over the urban area is 1000 m.

Normally temp. decreases with increase in altitude in the troposphere but not for the typical air pollution episodes. Those episodes are accompanied by a meteorological phenomenon called 'temperature inversion'. Under such condition, a layer is formed above the ground where temp. increases with increase in altitude. For such episodes, the inversion layer height is about 500 m.

Now, the 1st step is to identify the sources of pollutants

Sources:
(i) Direct emission to the atmosphere from the sources within the city.
(ii) Influx of pollutants from other areas upwind of the city.

The 2nd step is to identify the sinks of pollutants

Sinks:
(i) Pollutant deposition onto the ground within the city.
(ii) Atmospheric flow out of the city carried by the wind.

So, mass balance equation for the pollutants can be written as

(Mass emission rate into the urban air vol.)
+
(Mass ~~emission~~ flow rate into the vol. with the wind)
= (Mass removal rate onto the ground)
+
(Mass flow ~~out~~ rate out of the vol. with the wind)

Let 'C' be the pollutant conc. over the city in units of mass per vol. of air (kg/m³).

'M' be the mass emission rate in unit of mass per time

'R' be the mass removal rate in unit of mass per unit time

'C₀' be the background conc. of pollutants just outside the urban area in unit of mass per vol. of air.

Mass flow rate of pollutants into the vol. is proportional to the wind velocity 'u', cross-sectional area of the box perpendicular to the wind direction & background pollutant conc.

So, $M + u \Delta y H C^0 = R + u \Delta y H C$

or $M = R + u \Delta y H (C - C^0)$

$M = m \times \Delta x \times \Delta y$

where,

m = mass emission rate per surface area

$R = r \times \Delta x, \Delta y$

r = mass removal rate per surface area

& $r = v_d \cdot C$.

where, v_d = dry deposition velocity

C = conc. of species

$$m \times \Delta x \times \Delta y = v_d \cdot c \, \Delta x \times \Delta y + u \Delta y H (c - c^0)$$

or $m = v_d \cdot c + \frac{u}{\Delta x} H (c - c^0)$

or $\frac{m}{H} = v_d \frac{c}{H} + \frac{u}{\Delta x} (c - c^0)$

or $\frac{m}{H} = v_d \frac{c}{H} + \frac{c - c^0}{\tau}$ $\left[\because \left(\frac{u}{\Delta x}\right)^{-1} = \tau \right]$

or $\frac{m}{H} = \frac{v_d c}{H} + \frac{c}{\tau} - \frac{c^0}{\tau}$

or $\frac{m}{H} + \frac{c^0}{\tau} = c \left(\frac{v_d}{H} + \frac{1}{\tau} \right)$

or $\frac{m\tau + c^0 H}{H\tau} = c \left(\frac{v_d}{H} + \frac{1}{\tau} \right)$

or $\frac{m\tau + c^0 H}{H\tau} = c \left(\frac{v_d \tau + H}{H\tau} \right)$.

or $c = \frac{m\tau + c^0 H}{v_d \tau + H}$

Now, the rate of change of of mass within the box can be expressed mathematically as

$$\Delta x \Delta y H \cdot \frac{dc}{dt} = M - R + u \Delta y H c^0 - u \Delta y H c$$

or $$H \cdot \frac{dc}{dt} = \frac{M}{\Delta x \cdot \Delta y} - \frac{R}{\Delta x \Delta y} + \frac{H c^0 u}{\Delta x} - \frac{H c u}{\Delta x}$$

or $$H \cdot \frac{dc}{dt} = \frac{m \cancel{\Delta x \Delta y}}{\cancel{\Delta x \Delta y}} - \frac{v_d c \cdot \cancel{\Delta x \Delta y}}{\cancel{\Delta x \Delta y}} + \frac{H u}{\Delta x}(c^0 - c)$$

$[\because M = m \Delta x \Delta y$ & $R = r \Delta x \Delta y = v_d c \Delta x \Delta y]$

or $$H \cdot \frac{dc}{dt} = m - v_d c + H \frac{u}{\Delta x}(c^0 - c)$$

or $$\frac{dc}{dt} = \frac{m}{H} - \frac{v_d c}{H} + \frac{c^0 - c}{\tau} \qquad \left[\because \left(\frac{u}{\Delta x}\right)^{-1} = \tau\right]$$

or $$\boxed{\frac{dc}{dt} = \frac{m}{H} - \frac{v_d c}{H} - \frac{u}{\Delta x}(c - c^0)}$$

This is the differential equation for the average conc. within the box. & is derived from the consideration of mass conservation.