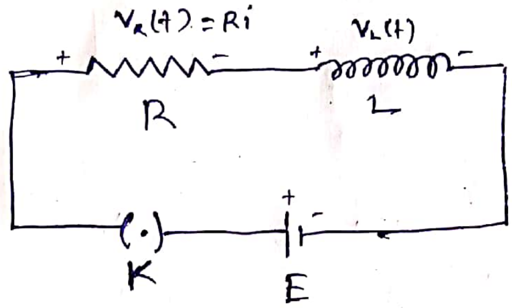
Subject name: Physics ; Semester - IV

Name of the teacher: Bidhan Chandra Jana

Name of topic: Transient Response of circuits

Series RL Circuit

Growth of Current -



Let $i(t)$ be the current in the circuit at any time t after closing the switch K. The instantaneous induced emf in the coil become $L\frac{di}{dt}$, and this will oppose the emf E. The net voltage driving the current i is $\left(E - L\frac{di}{dt}\right)$ and this must be equal to Ri.

$$\therefore \quad E - L\frac{di}{dt} = Ri$$

$$\Rightarrow \quad L\frac{di}{dt} + Ri = E \quad \cdots\cdots (1)$$

This is the emf equation of the circuit.

Now, from eqn. (1)

$$L\frac{di}{dt} = E - Ri$$

$$\Rightarrow \quad \frac{di}{E - Ri} = \frac{dt}{L}$$

Integrating,

$$\int_0^i \frac{di}{E - Ri} = \int_0^t \frac{dt}{L} \quad \left[\because \text{ at } t = 0,\ i = 0\right]$$

$$\Rightarrow \left[-\frac{L}{R}\ln(E-Ri)\right]_0^i = \left[\frac{t}{L}\right]_0^t$$

$$\Rightarrow -\frac{1}{R}\left[\ln(E-Ri) - \ln(E)\right] = \frac{t}{L}$$

$$\Rightarrow \ln\left(\frac{E-Ri}{E}\right) = -\frac{Rt}{L}$$

$$\Rightarrow 1 - \frac{R}{E}i = e^{-\frac{Rt}{L}}$$

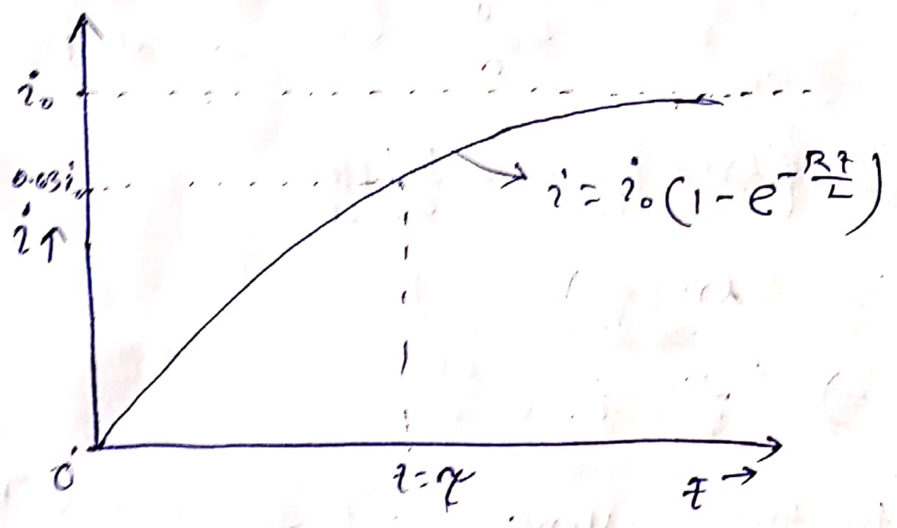
$$\Rightarrow \boxed{i = \frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right)} \quad \left[\frac{E}{R} = i_0 = \text{Steady state current}\right]$$

The quantity $\frac{L}{R}$ has the dimension of time and is called time constant (τ).

$\therefore$ at $t = \tau$,

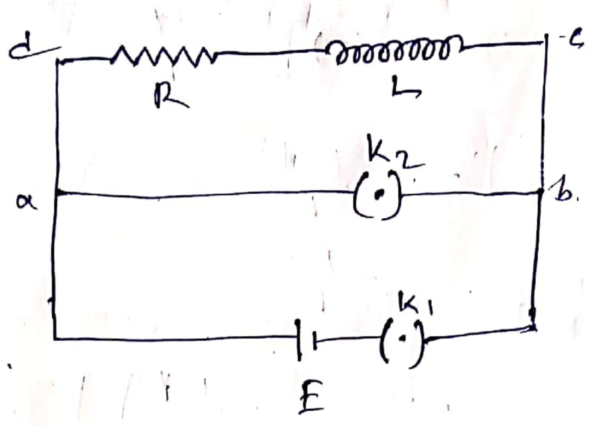
$$i = \frac{E}{R} 0.63 = 0.63\, i_0$$

$\therefore$ Time required to reach the 63 percent of its maximum value is time constant of the circuit.



Growth of current in RL Circuit

Decay of current



Once the circuit ~~at~~ reached to the steady state, at time $t=0$ the switch K_1 is opened and simultaneously the switch K_2 is closed. In the closed circuit abcd, the driving emf is zero,

Now, if $i(t)$ is the current at time t in the abcd, then,

$$L\frac{di}{dt} + Ri = 0$$

$$\Rightarrow \frac{di}{i} = -\frac{R}{L}dt$$

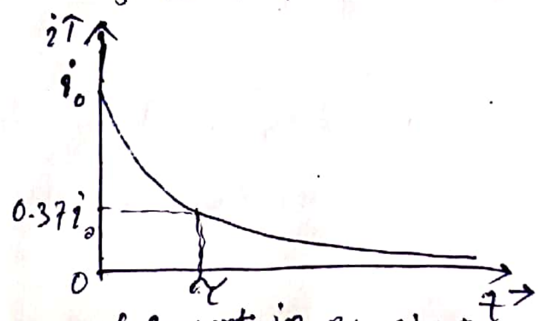
$$\Rightarrow \int_{i_0}^{i}\frac{di}{i} = -\frac{R}{L}\int_0^t dt \quad \left[\because \text{ at } t=0,\ i=i_0 \text{ and at time } t, \text{ current is } i\right]$$

$$\Rightarrow \left[\ln i\right]_{i_0}^{i} = -\frac{R}{L}\left[t\right]_0^t$$

$$\Rightarrow \ln\left(\frac{i}{i_0}\right) = -\frac{Rt}{L}$$

$$\Rightarrow \boxed{i = i_0 e^{-\frac{Rt}{L}}}$$

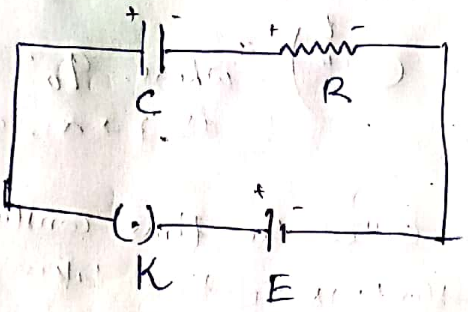
Here the time constant $\frac{L}{R}$ is the time taken by the current to decay 37 per cent of its initial value.



Decay of current in RL circuit.

Series RC Circuit

Charging of the Capacitor



Let, at any instant of time t, the voltage across the Capacitor be $v_c(t)$ and that across the resistance be $v_R(t)$

The emf equation for the circuit is

$$v_c(t) + v_R(t) = E \quad \text{--- (1)}$$

Since, $i = \frac{dq}{dt}$ and $q = C v_c$ = charged stored on the Capacitor at time t,

$$\therefore \; v_c(t) = \frac{q}{C} \quad \text{and} \quad v_R(t) = Ri = R\frac{dq}{dt}$$

From equn (1) —

$$\frac{q}{C} + R\frac{dq}{dt} = E$$

$$\Rightarrow \frac{dq}{E - \frac{q}{C}} = \frac{dt}{R}$$

Integrating —

$$\int_0^q \frac{dq}{E - \frac{q}{C}} = \int_0^t \frac{dt}{R}$$

$$\Rightarrow \left[-C\ln\left(E - \frac{q}{C}\right)\right]_0^q = \frac{1}{R}\left[t\right]_0^t$$

$$\Rightarrow \ln\left(E - \frac{q}{C}\right) - \ln(E) = -\frac{t}{CR}$$

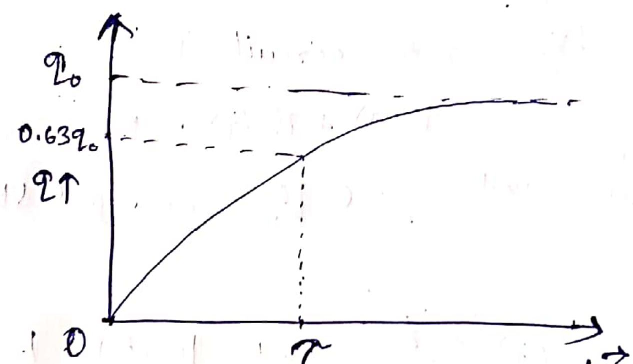
$$\Rightarrow \ln\left(\frac{E - \frac{q}{C}}{E}\right) = -\frac{t}{CR}$$

$$\Rightarrow 1 - \frac{q}{CE} = e^{-\frac{t}{CR}}$$

$$\Rightarrow q = CE\left(1 - e^{-\frac{t}{CR}}\right)$$

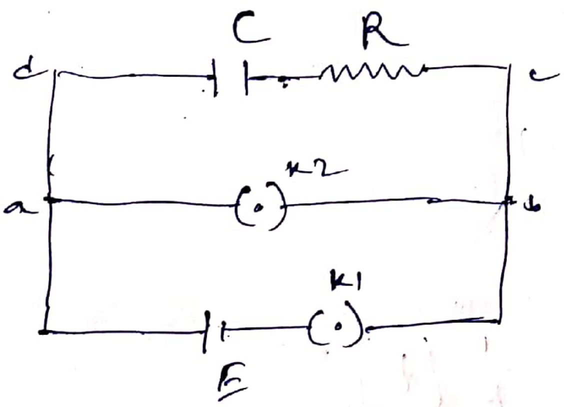
$$\Rightarrow q = q_0\left(1 - e^{-\frac{t}{CR}}\right)$$ [where, $q_0 = CE$ as $t \to \infty$, $q = q_0$]

Here, the quantity CR is called the time constant τ of the circuit. The time constant is the time taken for a capacitor to acquire 63 percent of its final maximum charge.



Variation of capacitor charge with time during charging.

Discharging of the capacitor –



Once the capacitor is fully charged to q_0, the switch K_1 is opened and K_2 is closed simultaneously.

In the closed path 'ancd', the driving emf is zero. So the emf equation is

$$\frac{q}{C} + R\frac{dq}{dt} = 0$$

$$\Rightarrow \frac{dq}{q} = -\frac{dt}{CR}$$

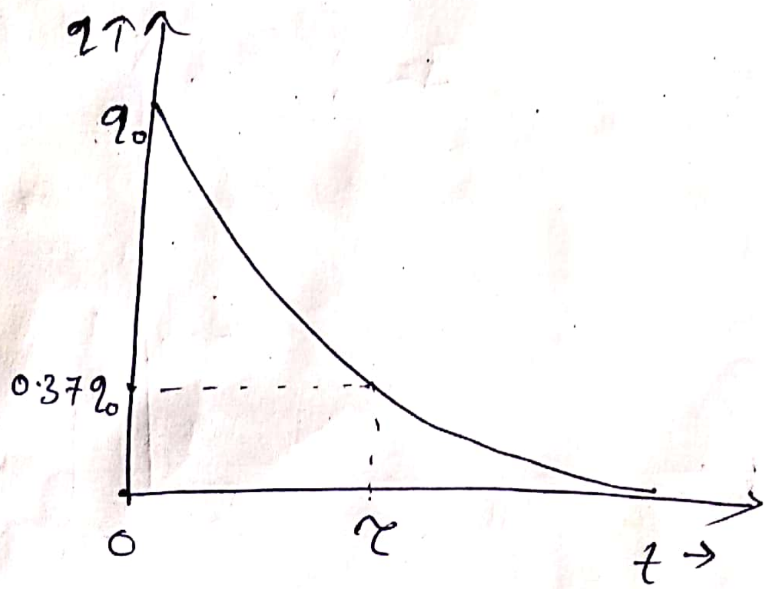
Integrating —

$$\int_{q_0}^{q} \frac{dq}{q} = -\frac{1}{CR}\int_0^t dt \quad \left[\because \text{ at } t=0,\ q=q_0\right]$$

$$\Rightarrow \left[\ln q\right]_{q_0}^{q} = -\frac{1}{CR}\left[t\right]_0^t$$

$$\Rightarrow \ln\frac{q}{q_0} = -\frac{t}{CR}$$

$$\Rightarrow \boxed{q = q_0 e^{-\frac{t}{CR}}}$$



Decay of capacitor charge with time.