Nuclear Fission

Nuclear fission is a special type of nuclear reaction in which an excited compound nucleus breaks up generally into two fragments of comparable mass numbers. Fission usually occurs amongst the isotopes of the heaviest elements known. eg. Uranium, Theorium etc.

Fission was discovered by Otto Hahn and F. Strassmann in 1939. While observing the effect of slow neutron bombardment on Uranium they tried to find out the radioactiactive products and found one of the product is Barium. Meitner and Otto Frisch provided the correct explan of this result by suggesting that the Uranium nucleus bombarded with neutron broke up into two large fragment. They gave the name nuclear fissi to this phenomenon.

Since the atomic number of Barium is 56, the other fragment produced in fission of Uranium should have the atomic number (92 - 56) = 36, the nucleus of isotope of krypton. The two fragments produced in fission are known as fission fragment

$$ {}^{235}_{92}U + {}^{1}_{0}n_{\text{(slow)}} \longrightarrow {}^{236}_{92}U^{*} \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3\,{}^{1}_{0}n \text{ (fast)} $$

which is highly exoergic.

Energy release in fission.

Nuclear fission is a highly exoergic reaction. Two Fission fragments carry. 167 Mev of energy. Some energy is also carried by γ rays and a few prompt neutrons emitted along with the fragments during fission. Besides these energies are also carried by β^- particle, antineutrinos and γ rays emitted by fission fragments.

Components	Energy (MeV)
K.E. of fission fragments	167.
K.E. of prompt neutrons	5.
Energy of prompt γ rays	6
Energy of β^- particles emitted by fission fragments	8
Energy of antineutrinos emitted by fission fragments.	12
Energy of γ rays emitted by fission fragments.	6
	204

The energy released during nuclear fission can be measured by bombarding a piece of Uranium with thermal neutrons, which is found to be heated due to absorption of fission fragments and some other products. The heat first generated is about 186 MeV per Uranium nucleus which is less than 204 MeV. This is due to the fact that the antineutrinos and γ rays produced have very high penetrability and hence escape from the Uranium piece.

from binding fraction curve, A heavy nucleus like uranium has a value of $f_B = B/A = 7.6$ MeV per nucleon. The fragments produced in its fission have mass numbers near the middle of the periodic table and hence the f_B value is 8.5 MeV/nucleon. Thus during the fission process, about 0.9 MeV energy per nucleon is released, so the total energy released is around $238 \times 0.9 = 212$ MeV.

$$Q = M(^{235}U) + M_n - M(^{141}Ba) - M(^{92}Kr) - 3M_n$$

$$= 200.6 \text{ MeV}$$

Energies of more or less of the same order of magnitude are released in the fission of other nuclei.

The number of atoms of ^{235}U per Kilogram is,

$$n = \frac{6.02 \times 10^{23} \times 10^{3}}{235} = 2.564 \times 10^{24}$$

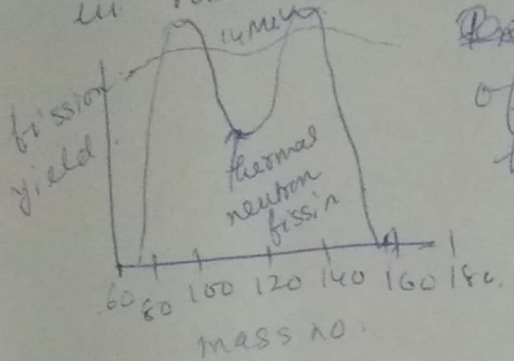
Hence the energy release per gram of ^{235}U is

$$E = \frac{nQ}{10^3} = \frac{2.564 \times 10^{24}}{10^3} \times 200.6 \times 1.6 \times 10^{-13}$$

$$= 2.29 \times 10^{4} \text{ kWh.}$$

Nature of fission fragments

Mass numbers of fission fragments are not same in most cases. There is a certain spread in the distribution of mass number. In case of thermal neutron fission of 235 U, the lighter fragments have the mass number spread between 85 to 105 with a broad peak at A ~ 96. The heavier fragments have the mass number distributed betn. 130 to 150 with a broadpeak at A ~ 138. Thus the fission is assymetric in this case.



- If the excitation energy of the compound nucleus undergoing fission is increased (higher neutron energy) symmetric fission becomes more probable. Symmetric fission is also more probable when fission is induced by particles other than neutron.
- The fission fragments are highly neutron rich. The two fission fragments 141 Ba and ^{92}Kr have the neutron excess $N - Z = A - 2Z = 29$ and 20 respectively, so that they must be β active.

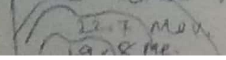
3.

$$^{141}Ba \xrightarrow[18\ min]{} {}^{141}La \xrightarrow[3.7\ h]{} {}^{141}Ce \xrightarrow[33\ d]{} {}^{141}Pr \text{ (stable)}$$

$$^{92}Kr \xrightarrow[3\ s]{} {}^{92}Rb \xrightarrow[80\ s]{} {}^{92}Sr \xrightarrow[2.7\ h]{} {}^{92}Y \xrightarrow[3.5\ h]{} {}^{92}Zr \text{ (stable)}$$

When the energy of bombarding neutron increases symmetric fission is more probable and two peak gradually merge.

45.7 MeV



Energy Distribution betn. the fission fragments

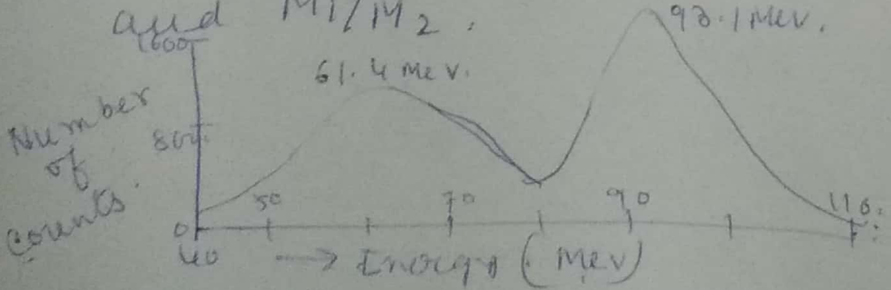
The asymmetry in the mass distribution of the fission fragments are reflected, in the distribution of the K.E. between them.

K.E. of the fission fragments have been measured using ionization chamber within which a thin foil of U or Th is placed. The ions produced by the two fragments that appear simultaneously in the two halves of the chamber are collected in the electrostatic field and ionization currents measured by two two electrometers calibrated by α particles of known energy. It is generally assumed that the energy required to produce an ion pair in argon is same for both the fission fragments as well as for the α particles and total energy is proportional to the total ionization current.

From conservation of momentum $M_1 v_1 = M_2 v_2$
where M_1 and M_2 are masses of two fission fragments.
Hence, the ratio of energies,

$$\frac{E_1}{E_2} = \frac{M_1 v_1^2}{M_2 v_2^2} = \frac{M_2}{M_1}$$

It is thus possible to determine the total energy (E_1+E_2) for each fission, as also the ratios E_1/E_2 and M_1/M_2.

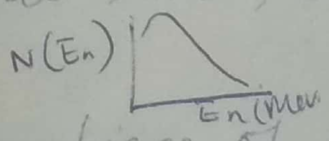


Energy distribution of fission fragments

Emissions of neutrons in nuclear fission.

The fission fragments, which are produced the breakup of the compound nucleus (e.g. ^{236}U should be such that their mass numbers add to the mass number of the fissioning nucleus (2 They are called primary fragments. Because the large neutron excess in them, one or both them, generally emit a few neutrons to reduce neutron excess to some extent. These fragments finally reach to stable products after β^- decay.

- ~99% neutrons are emitted within possibly (energy) sec and are called prompt neutrons. They follow M distribution with energy range 0.05 MeV to 17 MeV with peak at 2 MeV.
- A smaller number <1% of neutrons are em within 10^{-20} s to more than a minute and are called delayed neutrons. Their emissio follows the exponential law, as in the case of radioactive decay.



- For sufficiently large piece of fissionable subst the neutrons that are released in a first fission proce will be absorbed by the other nuclei and produce new processes which in turn emit new neutrons. Some of the compound nuclei decay to ground sta by gamma emission rather than fission.

$$\alpha = \frac{\sigma_r}{\sigma_f} = \frac{\text{radiative capture cross section}}{\text{fission cross section}}$$

the no of fission neutron released per neutron absorbed in fissionable nucleide $\eta = \frac{\nu_{av}}{(1+\alpha)}$ where ν_{av} is the avg no of neutrons released per fission

Slow neutron: those with K.E. $T_n < 1\,eV$.

Thermal neutron $T_n \rightarrow 0.025\,eV \sim 25\,MeV$.

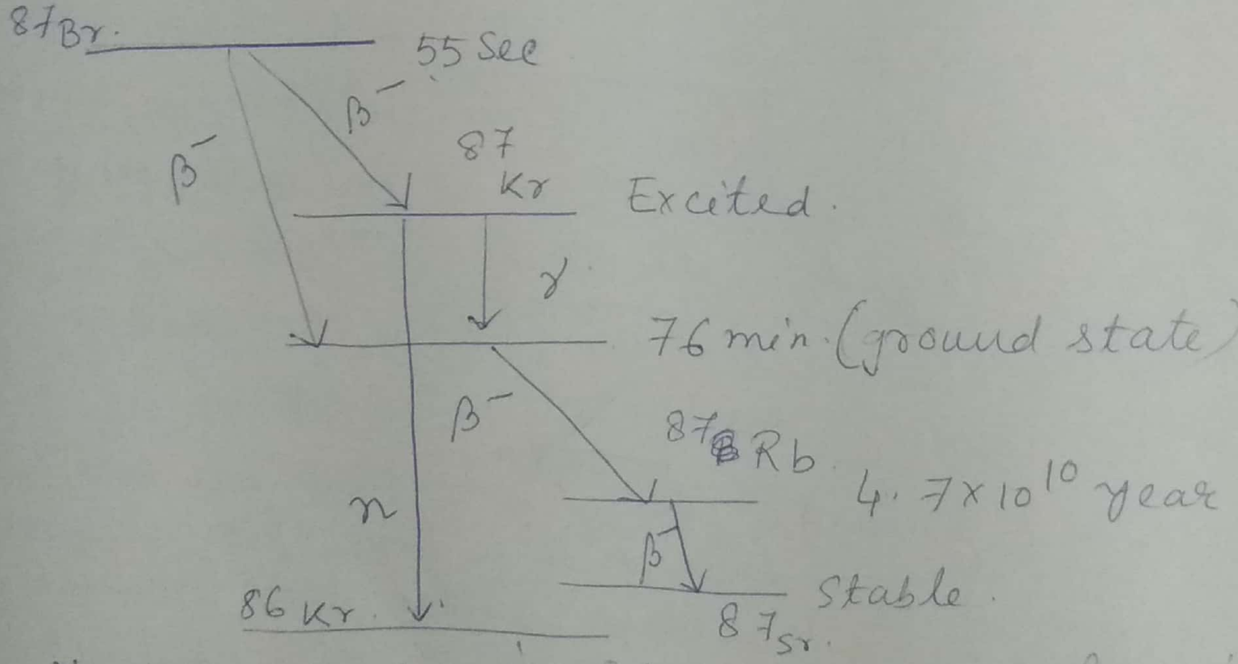
epithermal neutron. $1\,eV < T_n < 100\,keV$ (0.1 MeV)

fast neutron: $0.1\,MeV < T_n < 20\,MeV$.

~~$n + {}^{235}_{92}U \rightarrow {}^{236}_{92}U \rightarrow {}^{87}_{35}Br + {}^{146}_{57}La + 3\,{}^{1}_{0}n$~~ (2% con

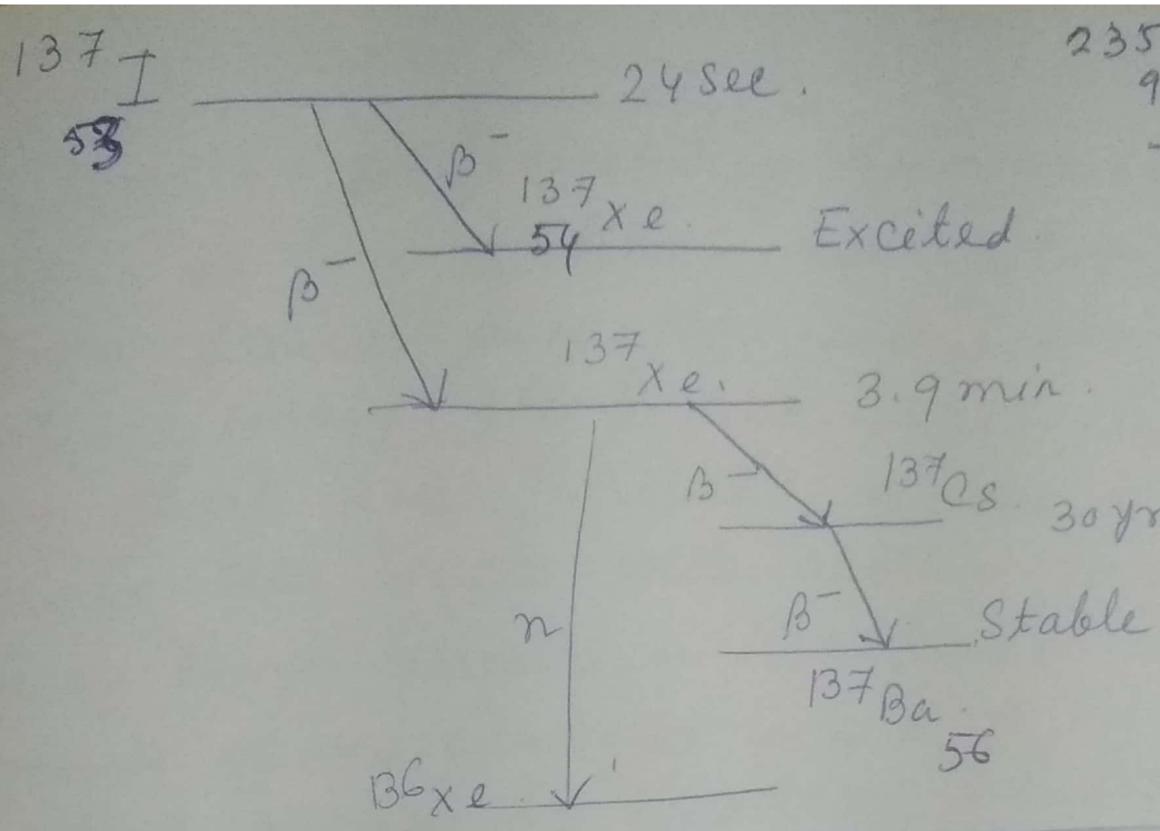
$n + {}^{239}_{94}Pu \longrightarrow {}^{87}_{35}Br + {}^{150}_{59}Pr + ?\,n.$

${}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{236}_{92}U \rightarrow {}^{148}_{57}La + {}^{85}_{35}Br + 3\,{}^{1}_{0}n$



Neutron number in ^{87}Kr is $N = 51$ which is reduced to magic number $N = 50$ in 86 Kr. formed by neutron emission from the former.

- The emission of the prompt neutrons in nuclear fission makes possible the realization of chain reaction.
- ^{235}U and ^{239}Pu are easier to fission than ^{238}U. as nuclear force is more attractive for an even number of neutrons in a nucleus compared to odd number.



$^{235}_{92}U + ^{1}_{0}n \rightarrow$ 2

$\rightarrow ^{137}_{52}Te +$ 9

+2

$^{137}_{52}Te$ β

137

53

light Nucleus :— Electrical forces are small compared to forces of surface tension.

Critical energy to seperate two parts (where two fragments are connected by a very narrow neck of liquid), is given by the diff in energy betn. the original nucleus and total energy of the fragments just separated.

$$E_0 = E = (E_S + E_C)_0 = a_s A^{2/3} + a_c \frac{Z^2}{A^{1/3}}$$

$$= 4\pi r_0^2 A^{2/3} \xi + \frac{3}{5} \frac{Z^2 e^2}{4\pi\epsilon_0 r_0 A^{1/3}}$$

where ξ is surface energy/unit area.

Energy after separation

$$E = 2a_s \left(\frac{A}{2}\right)^{2/3} + 2a_c \frac{(Z/2)^2}{(A/2)^{1/3}} + \frac{(Ze/2)^2}{4\pi\epsilon_0 \times 2r_0 \left(\frac{A}{2}\right)^{1/3}}$$

↓.

electrostatic energy betn. the two fragments when they are separated out, the distance betn their centres are $2R'$ where $R' = r_0 \left(\frac{A}{2}\right)^{1/3}$

$$= 2 \times 4\pi r_0^2 \left(\frac{A}{2}\right)^{2/3} \xi + \frac{1}{4\pi\epsilon_0} \left\{ 2 \times \frac{3}{5} \frac{\left(\frac{Ze}{2}\right)^2}{r_0 \left(\frac{A}{2}\right)^{1/3}} + \frac{\left(\frac{Ze}{2}\right)^2}{2r_0 \left(\frac{A}{2}\right)^{1/3}} \right\}$$

∴ Critical energy of deformation to cause fission.

$$\Delta E_{cr} = E - E_0.$$

$$= 4\pi r_0^2 A^{2/3} \xi (2^{1/3} - 1) + \frac{1}{4\pi\epsilon_0} \frac{3}{5} \frac{(Ze)^2}{r_0 A^{1/3}} \left(\frac{2^{1/3}}{2} - 1 + \frac{5 \cdot 2^{1/3}}{3.8} \right.$$

writing $y = \frac{\Delta E_{cr}}{4\pi r_0^2 A^{2/3} \xi}$

and $x = \frac{3Z^2 e^2}{2 \times 4\pi\epsilon_0 \times 5 r_0 A^{1/3} \times 4\pi r_0^2 A^{2/3} \xi}$

$$= \left(\frac{Z^2}{A}\right) \cdot \frac{3e^2}{4\pi\epsilon_0 \times 40 \pi r_0^3 \xi}$$

$y = 0.26 - 0.215 \cdot x$.

So for light nuclei, the critical energy of deformation for causing fission is a linear fn. of $\left(\frac{Z^2}{A}\right)$

Heavy nucleus:- The electrostatic forces within nucleus (Bohr Wheeler theory) play a predominant role and even a slight deformation of the drop will tend to build up against the forces of surface tension.

Consider the nucleus is initially a spherical shape made of incompressible nuclear fluid of const volume. So the vibration set up in the nucleus cant go any deep and remain confined to the surface.

Assuming the deformation to have an axial symmetry.

$r(\theta) = R_0 \left[1 + \sum_{l=0}^{\alpha} \alpha_l P_l \cos\theta \right]$ where $r(\theta) \rightarrow$ distance of a pt on nuclear surface at polar angle θ.

$R_0 \rightarrow$ radius of original undeformed spherical nuclear droplet, $\alpha \rightarrow$ deformation parameter.

The splitting of two fragments is mainly dependent on $P_2(\cos\theta)$ not on $P_1(\cos\theta)$ as this term only give a bodily translation without any deformation.

$$E_{S_0} = \text{surface energy} = a_S A^{2/3} = 4\pi R_0^2 \tau A^{2/3}$$

$$E_{C_0} = \text{Coulomb energy} = a_C Z^2/A^{1/3} = \frac{1}{4\pi\epsilon_0} \frac{3Ze^2}{5 r_0 A^{1/3}}$$

$$E_S = 4\pi R_0^2 A^{2/3} \tau \left[1 + \alpha_2 \left(\frac{3}{2}\cos^2\theta - \frac{1}{2} \right) + \cdots \right]^2$$

$$= 4\pi R_0^2 A^{2/3} \tau \left[1 + \frac{2}{5}\alpha_2^2 + \frac{7}{5}\alpha_3^2 + \cdots \right]$$

$\therefore \Delta E_S$ = Change in surface energy

$$= E_S^{\text{sphere}} \left[\frac{2}{5}\alpha_2^2 + \frac{5}{7}\alpha_3^2 + \cdots \right.$$

The coulomb energy $E_{C_0} = \frac{3}{5} Z^2 e^2 / 4\pi\epsilon_0 R$.

$$= \frac{3}{5} \frac{Z^2 e^2}{4\pi\epsilon_0 R_0 A^{1/3}} \left[1 + \alpha_2 \left(\frac{3}{2}\cos^2\theta - \frac{1}{2} \right) + \cdots \right]^{-1}$$

$$\therefore \Delta E_C = E_C \left[-\frac{1}{5}\alpha_2^2 - \frac{10}{49}\alpha_3^2 + \cdots \right]$$

$\therefore \Delta E = \Delta E_S + \Delta E_C = (E_S - E_{S_0}) + (E_C - E_{C_0}) =$ [illegible]

$$= 4\pi R_0^2 S A^{2/3}\left\{\frac{2}{5}\alpha_2^2 + \frac{116}{105}\alpha_2^3 + \frac{101}{35}\alpha_2^4 + \frac{2}{35}\alpha_2^2\alpha_4 + \alpha_4^4 + \dots\right]$$

$$-\frac{1}{4\pi\epsilon_0}\frac{3}{5}\frac{(Ze)^2}{r_0 A^{1/3}}\left\{\frac{1}{5}\alpha_2^2 + \frac{64}{105}\alpha_2^2 + \frac{58}{35}\alpha_2^4 + \frac{8}{35}\alpha_2^2\alpha_4 + \frac{5}{27}\alpha_4^2 + \dots\right]$$

The min^m energy of deformation can $\longrightarrow$ ① be found by minimizing. ①. w.r.t α_4. which gives

$$\alpha_4 = -243\,\alpha_2^2/595$$

∵ α's are small numbers, the main contributions to the deformation energy. comes from the terms containing α_2^2

$$\Delta E = \alpha_2^2\left(\frac{2}{5}a_2 A^{2/3} - \frac{1}{5}a_3\frac{Z^2}{A^{1/3}}\right).$$

$$= \Delta E_S + \Delta E_C = \frac{1}{5}\alpha_2^2\left[2E_S - E_C\right]. \longrightarrow ②$$

If it is positive., i.e. $2E_S > E_C$.

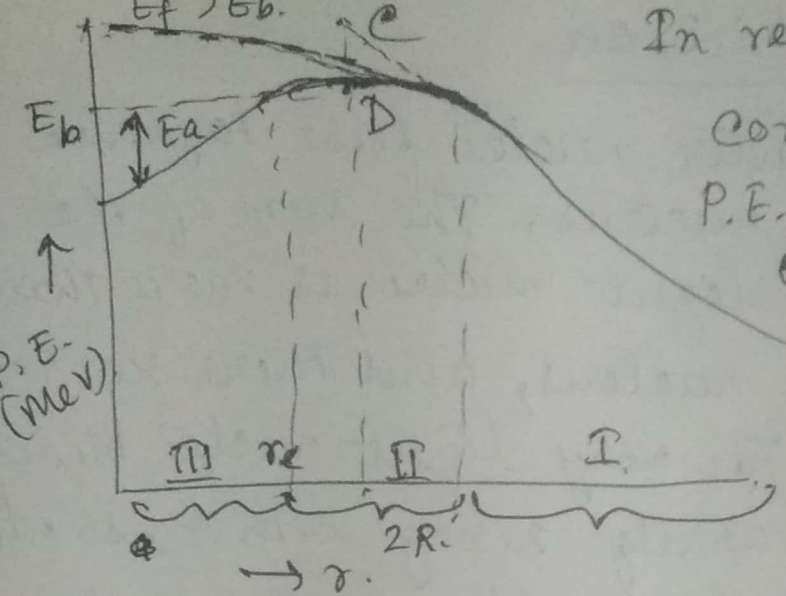
the drop is stable to small distortion. Fissions may occur spontaneously. if ΔE is negative $E_S < \frac{1}{2}E_C$.

$$4\pi R_0^2 A^{2/3} \tau < 3Z^2e^2/40\pi\epsilon_0 A^{1/3} R_0$$

$$\text{or,}\quad Z^2/A > 45.$$

The ratio $E_C/2E_S$ is known as critical parameter represented by x. Thus when $x > 1$. the nucleus is stable against spontaneous fission.

for an uncharged droplet $x = 0$ and hence there are no electrostatic forces and critical energy is just the work done against surface tension in separating into two drops.



P.E for fission.

In region I, the fragments are completely separated and their P.E. E is simply the electrostati coulomb energy resulting from the mutual repulsion of two positively charged nuclear fragments. If $r = 2R$, whe the drop just touch each other, Energy E at the pt. is less than the Corresponding coulomb potential by an amount CD. This is equal to the Potential of the surface forces which are just beginning to come into play at this pt. As we pass through region II, we reach the critical distance r_c where the P.E. curve has a max^m^ value E_b. This corresponds to barrier height and explains why fission does not take place spontaneously in all cas where $E_f > 0$. An additional amount of energy. $E_a = E_b - E_f$ = Activation energy is required by th nuclear system before the potential barrier can be surmounted and fission can take place. In regio III the fragments have coalesced and short rang nuclear forces have become predominant.

Spontaneous Fission:- Most heavy nuclides undergo spontaneous fission. It is predicted by the empirical nuclear mass equation.

Consider → nucleus splits into two equal parts.

neglecting δ, the pairing term, Q value for fission reaction,

$$E_f = \left[{}_Z M^{A} (Z, A) - 2 \left({}_{Z/2} M^{A/2} (A/2, Z/2) \right) \right] c^2 \rightarrow ①$$

considering mass formula,

$$M(Z, A) = Z M_p + (A - Z) M_n - a_v A + a_s A^{2/3} + a_c Z^2 A^{-1/3} + a_a (A - 2Z)^2 A^{-1} \rightarrow ②$$

$$M(\tfrac{1}{2}Z, \tfrac{1}{2}A) = \tfrac{1}{2} Z M_p - \tfrac{1}{2}(A - Z) M_n - a_v (\tfrac{1}{2}A) + a_s (\tfrac{1}{2}A)^{2/3} + a_c (\tfrac{1}{2}Z)^2 (\tfrac{1}{2}A)^{-1/3} + \tfrac{1}{2} a_a (A - 2Z)^2 A^{-1} \rightarrow ③$$

Substituting value of ${}_Z M^A$ and ${}_{Z/2} M^{A/2}$ in ①

$$E_f = \left[a_s \left\{ A^{2/3} - 2 (\tfrac{1}{2}A)^{2/3} \right\} + a_c \left\{ Z^2/A^{1/3} - 2 (\tfrac{1}{2}Z)^2 / (\tfrac{1}{2}A)^{1/3} \right\} \right] \times c^2$$

$$= -3.42 A^{2/3} + 0.22 Z^2 / A^{1/3} \text{ MeV}.$$

Since the division of nucleus increases. 1) the separation betn proton groups, thus reducing their Coulomb potential energy. (2) the total nuclear surface which increases the surface energy., i.e. the splitting of nucleus affects Coulomb energy and surface energy in such a way that the change in one and that in other tend to cancel one another partially.

∴ for spontaneous fission

$$-3.42 A^{2/3} + 0.22 \frac{Z^2}{A^{1/3}} \geq 0$$

$$\text{or, } Z^2/A \geq 15 \rightarrow ④$$

∴ It should be energetically possible for nuclei with mass number $A \geq 85$.

However, the slow neutron fission does not take place even with many of the heavy nuclei. Bohr and Wheeler considered Coulomb's potential barrier of the two fragments at the instant of separation. The existence of this barrier prevents the immediate breaking of these two.

The nucleus will be unstable and break apart into two fragments if $E_f > E_b$.

$$E_b = \frac{\left(\frac{1}{2}Z\right)^2 e^2}{4\pi\epsilon_0 (2R)} = \frac{Z^2 e^2}{32\pi\epsilon_0 R_0 \left(\frac{1}{2}A\right)^{1/3}}$$

$$= 0.15\, Z^2/A^{1/3} \text{ MeV.}$$

$$\therefore E_b - E_f = 0.15\, Z^2/A^{1/3} - \left[-3.42\, A^{2/3} + 0.22\, Z^2/A^{1/3}\right]$$

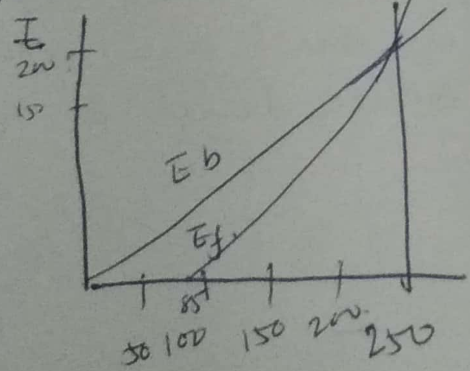
$$= 3.42\, A^{2/3} - 0.07 \frac{Z^2}{A^{1/3}}$$

∴ condition for stability gives,

$$E_b - E_f \geq 0$$

or, $\frac{Z^2}{A} \leq 49.$

For a particular nucleus, the closer the value of $\frac{Z^2}{A}$ to 50, the shorter the half life for spontaneous fission i.e. mass number $A > 250$. (10^{-12} sec or less)



$A > 250$

$E_f > E_b$

$A \approx 250$, $E_f = E_b$

Transuranic Elements :— The elements lying beyond $^{238}_{92}U$ in the periodic table are called transuranic element. These do not occur in nature but are man made.

The Capture of a neutron by Uranium does not necessarily lead to a fission of the compound nucleus. U-238 for example has a small capture cross section of low energy neutrons and leads to the formation of new elements as they lie beyond Uranium in the periodic system.

$$ {}_{92}U^{238} + {}_{0}n^{1} \longrightarrow {}_{92}U^{239} + \gamma $$

$$ {}_{92}U^{239} \longrightarrow {}_{93}Np^{239} + {}_{-1}e^{0} \quad (T = 23\,\text{min}) $$

e.g. Np → Neptunium. ${}_{93}Np^{231-241}$; Pu → Plutonium ${}_{94}Pu^{232-246}$ etc.

Nuclear Chain Reaction.

Chain reaction is a self sustaining process that once started needs no additional agents to keep it going.

In fission, to maintain a chain reaction, the neutrons must be produced at a const rate and number of neutrons ~~must be~~ ~~produced~~ released per fission ν must be sufficiently greater than 1 to compensate neutron loss

Let us consider a generation of neutron.

Natural U contains U^{235} & U^{238} with 0.72% and 99.28% abundances

- The chain reaction with U depends on favourable bala among these processes.

1) Neutron may be absorbed by U^{238} and may cause fission when its energy is greater than the threshold energy.

2) Neutron may be absorbed by U^{238} without producing fission.

3). Neutron may be absorbed by U^{235} nucleus causing fission.

4). Neutron may be absorbed by U^{235} without causing fission.

5). Neutron may be absorbed by other materials present in the assembly without causing fission

6). Neutron may escape without being captured. (Neutron leakage).

So processes (1) and (3) create new neutrons while other remaining processes remove available neutrons from the assembly.

- A device for releasing nuclear energy through fission at a controlled rate is called nuclear reactor or chain reacting pile

There should be critical size of the system, so that nuclear chain reaction is possible, the surface area is ~~do~~ responsible for neutron loss and vol is for neutr product So surface to volume ratio is a factor and if it is large chain reaction is not possib

- Nuclear Weapons → uncontrolled nuclear reaction is desirable.

- Thermal Reactor → Neutron balanced is required.

and it is described ~~by~~ in terms of a cycle that shows what happens to the neutrons.

- Consider → cycle starts with the fission of U^{235} nucleus by a thermal neutron. As a result of avg value 2.47 ν fast neutrons are emitted, some of them have energy greater than the fission threshold for U^{238} can cause fission in U^{238} with consequent increase of number of fast neutrons. So number of fast neutrons is increased from ν to $\nu\epsilon$ where ϵ is greater than one and is called fast fission factor.

The value of ϵ lies between 1.03 and 1.1. As these $\nu\epsilon$ fast neutrons move through the moderator system most of them will be slowed down du to collisions with Uranium and moderator nuclei but

From a finite sized reacter core, some fast neutrons may leak out before being slowed down to thermal energies

l_f → leakage facter. for fast neutrons. then the number of fast neutron leaking out = $\nu\epsilon l_f$

$\nu\epsilon(1-l_f)$ → number of fast neutrons remaining in core for slowing down to thermal energies.

These neutrons are slowed-down by collisions with moderator atoms, but during the slowing down process some of them may be captured by U^{238} to form U^{239} which decays by β^- emission to Np^{239} and then to Pu^{239}, particularly at some specific energies in the region of strong resonances (resonance capture).

Those neutrons which escapes from resonance capture are finally slowed down to thermal energies.

p → is resonance capture probability.

~~out of~~ $\nu\epsilon(1-l_f)(1-p)$ ~~is~~ ~~ca~~ is captured. and goes to form Pu^{239}, and $\nu\epsilon(1-l_f)p$ is escaped resonance capture. ↓ slowed down

l_s → leakage factor of thermal neutron

No of thermal neutron leaking away = $\nu\epsilon p(1-l_f)l_s$.

The number of slow or thermal neutrons remaining in the core is $\nu\epsilon(1-l_f)p(1-l_s)$.

A fraction of this remaining number is absorbed by U_{235}^{92} (nuclear fuel): and the remaining other part are absorbed in moderator or by other materials.

The fraction f of thermal neutrons which are absorbed in the fuel is called thermal utilization factor. So the ~~final number of second generation~~ number of thermal neutron actually ~~neutron~~ absorbed in the fuel is $\nu\epsilon(1-l_f)p(1-l_s)f$

~~This quantity is known as reproduction factor or multiplication factor k~~ =

If σ_a is the cross section of thermal neutrons absorbed in U^{235}

" σ_f " " " cause fission of U^{235}

Of these neutrons only a fraction $\frac{\sigma_f}{\sigma_a}$ induces fission.

So the final number of second generation neutron absorbed in the fuel is $\nu\epsilon(1-l_f)p(1-l_s)f\frac{\sigma_f}{\sigma_a}$.

The quantity $\nu\left(\frac{\sigma_f}{\sigma_a}\right) \equiv$ the number of fast fission neutrons produced by each thermal neutrons. absorbed by the fuel.

$\equiv \eta$.

$\therefore k = \eta\epsilon(1-l_f)\,p\,(1-l_s)\,f/1 =$ reproduction factor / multiplication factor.

$= \frac{\text{no of second generation neutron inducing fission in U}_{235}}{\text{no of first gen neutron inducing fission in U}_{235}}$

If the core is of very large size (i.e. chain reaction system is very large). the leakage of fast as well as slow neutrons can be disregarded. i.e. negligibly small.

then $k \equiv k_\infty = \eta\epsilon p f$. which is known as <u>four factor formula</u>.

In an infinite system, the conditions. of the self-sustaining chain reaction is that each neutron generation just replaces the previous one. i.e. $k=1$

The system is then said to be <u>critical</u>

when this cond is fulfilled. then neutron density n(t)

for finite system $k > 1$. $\frac{dn}{dt} = \frac{n(k-1)}{\tau}$ where τ is generation time.

$\therefore n(t) \overset{\tau}{=} n(0)\,e^{(k-1)t/\tau}$ but actually neutron d is controlled by cadm rod is ve slow n absorb

for infinite system if $k > 1 \rightarrow$. super critical state.

$k < 1 \rightarrow$. sub critical state.

- when fuel contains U_{235} only and no U_{238} – both the factors ϵ and p are practically unity.

The formula. for $k_\infty = \eta f$.

For an assembly of finite dimensions, the effective multip factor k_{eff} will be less than k_∞ by a factor $(1-l_f)(1-l_s)$

thus $k_{eff} = k_\infty(1-l_f)(1-l_s)$ The magnitude of k_{eff} determines the speed with which the no of neutrons builds up and the rate at which fission occurs. In a nuclear bomb type of assembly the build up take place rapidly ($k_{eff} > 1$), whereas in industrial and research reactor. $k_{eff} \simeq 1$. (it is controlled) If $k_{eff} > 1$, the reactor size can be decreased progressively, so neutron loss through leakage is increa

The reactor size for which $k_{eff} = 1$, is called critical size

General Features of Reactors:-

1) Fuel:- The material containing the fissile isotope is called the reactor fuel. The fuels that can be used are Uranium containing U^{235} in its natural concentration of 0.715%. or in an enriched proportion. The remaining 99.3% being ^{238}U is called Fertile mat
It canbe converted to fissile ^{239}Pu $^{238}U + ^{1}_{0}n \rightarrow ^{239}U \xrightarrow[23.5m]{\beta^-} ^{239}Np \xrightarrow[2.35d]{\beta^-} ^{239}P$ which is called conversion.

2) Moderators and Reflectors:- Materials used to reduce the neutron energy known as moderators. are graphite, light water(H_2O) heavy water(D_2O), Beryllium and its oxide and certain organic compound, good moderator are usually have low mass number, having small absorption cross-section and large slowing down power. The neutrons are slowed down by elastic collisions with the moderator nuclei.

Most reactors use reflectors around them to reflect back ~~to~~ the neutrons leaking out ~~to~~ of the reactor. Materials for reflectors should be same as of moderators. For best result the moderator nuclei should be of same size as neutrons and capture cross section of the moderator for neutrons must be as small as possible. So hydrogeneous materials are best suited as moderators. Thus instead of water, heavy water is used as it has small capture cross section

3) Reactor Coolants:- The materials employed, to remove the heat that is being generated in the reactor core as a result of fissioning taking place, are known as coolants.

Ideal coolant should ~~have~~ not absorb. nor moderate the neutrons, should not react chemically with other material, should not break up under irradiation, should acquire intense long lived radioactivity during its passages through reactor, should have low vapour pr, should be able to remove large amount of heat

e.g. Liquid Coolant :- ordinary water, heavy water, mercury liquid sodium etc, Gas Coolant :- Air, Nitrogen, Helium and C

Control material :- Cadmium or alloy of Silver with 15% of indium and 5% of Cadmium. These are of high neutron absorption cross section.

Control is achieved by means of a neutron absorbing materials. The materials should not be radioactive after neutron capture. It is co located i the core in form of rods or plates, but in some reactors it is more convinent to ha the control elements in the reflector close to core. It is used to decrease the fission rat

Reactor shielding: layer of Concrete sheet surrounding the reactor which is 6 to 8 ft thick capable of absorbing both γ ray and neutrons

Purpose of Reactor:- 1) Reseach 2) Thermal / Electric power production

Nuclear Fusion

In fusion process, lighter nuclei fuse together and produce a heavier nucleus. The sum of the masses of the individual light nuclei is more tha the mass formed by the nucleus, and there resu a liberation of energy. For very light nuclei bin energy per nucleon f_B is rapidly rising with A, so a nucleus is produced by fusion of two lighter nuclei may have greater binding energy. Bethe suggested that the process of stellar energy is by thermonuclear rea in which He-4 nuclei are synthesized from 4 protons.

1) Carbon nitrogen cycle (major part of Sun's energy) comes from this cycle

2) p-p cain (responsible for Central temp of sun).

fusion :- Carbon Nitrogen cycle :- Carbon acts as a catalyst in facilitating the combination of fou protons to form a helium ~~nu~~ nuclide. First, a pro ~~first~~ interacts with 12C nucleus with a release of fusion energy. The product nucleus 13N decays in very short time. The stable nucleus of 13C, thus formed then reacts with another proton. The mas energy being liberated in this process. The stable nucleus of 14N combines with a third proton. Th product nucleus 15O is a (+ve) β emitter, which decays into 15N. This nucleus finally interacts with a fourth proton and regenerates 12C nucle This chain of reactions can start with either carbon or nitrogen, since each one is reproduced in the reaction. The four protons are associated with four electrons to maintain electrical neutrality, wherea only two are required for helium nucleus, and rest two electrons combine readily with positrons resulti in the formation of γ-rays.

The mass difference released as energy in this chain reactions is simply the difference between th masses of four protons and one helium nucleus. A. Small amount of this energy is carried away by. the neutrons that are emitted during the e^+ decay. Bethe estimated this to be about 1.84 MeV leaving rest for each α-particle formed.

$_6C^{12} + {}_1H^1 \rightarrow ({}_7N^{13}) \rightarrow {}_7N^{13} + 1.94$ MeV (10^6 yr)

decay $_7N^{13} \rightarrow ({}_6C^{13}) + {}_{+1}e^0 + \nu + 1.20 + 1.02$ MeV (10min

$_6C^{13} + {}_1H^1 \rightarrow ({}_7N^{14})^* \rightarrow {}_7N^{14} + 7.55$ MeV (2×10^5 yr)

$_7N^{14} + {}_1H^1 \rightarrow ({}_8O^{15}) \rightarrow {}_8O^{15} + 7.29$ MeV (3×10^7 yr)

$_8O^{15} \rightarrow {}_7N^{15} + {}_{+1}e^0 + \nu + 1.74 + 1.02$ MeV (2 min)

$_7N^{15} + {}_1H^1 \rightarrow ({}_8O^{16}) \rightarrow {}_6C^{12} + {}_2He^4 + 4.96$ MeV (10^4 yr

Altogether:- $4\,{}_1H^1 \rightarrow {}_2He^4 + 2\,{}_{+1}e^0 + 2\nu + 26.7$ MeV

This chain reaction can start with either Carbon or Nitrogen since each one is reproduced in the reaction. The four protons are associated with four electrons to maintain electrical neutrality whereas two only are required for the helium nucleus and rest two electrons combine readily with protons resulting in the farmation of γ rays.

P-P Chain reaction:- Two protons first fuse to produce a deuterium which combines with another proton to form 3He. Then two 3He combine and produce 4He and two protons.

$2\times[{}_1H^1 + {}_1H^1 \rightarrow {}_2He^2 \rightarrow {}_1H^2 + {}_{+1}e^0 + \nu + 0.42$ MeV (7×10^9 yr)]

$2\times[{}_1H^2 + {}_1H^1 \rightarrow {}_2He^3 \rightarrow {}_2He^3 + \gamma + 5.5$ MeV (10 sec)]

$_2He^3 + {}_2He^3 \rightarrow ({}_4Be^6) \rightarrow {}_2He^4 + {}_1H^1 + {}_1H^1 + 12.86$ MeV (3×10^5 yr)

$4\,{}_1H^1 \rightarrow {}_2He^4 + 2\,{}_{+1}e^0 + 2\nu + 2\gamma + 26.7$ MeV → (I type)

p-p chain :- ⊛

With 4He present in the core of the star, either generated by hydrogen burning or present in the original formation, there are other possible paths in the hydrogen burning chain after formation of 3He.

$${}^{3}_{2}He + {}^{4}_{2}He \rightarrow {}^{7}_{4}Be + \gamma$$

followed by either,

$${}^{7}_{4}Be + {}^{0}_{-1}e \rightarrow {}^{7}_{3}Li + \gamma.$$

$${}^{7}_{3}Li + {}^{1}_{1}H \rightarrow + {}^{4}_{2}He + {}^{4}_{2}He$$ → Path-II.

or,

$${}^{7}_{4}Be + {}^{1}_{1}H \rightarrow {}^{8}_{5}B + \gamma$$

$${}^{8}_{5}B \rightarrow {}^{8}_{4}Be + {}^{0}_{+1}e + \gamma$$

→ Path-III

$${}^{8}_{4}Be \rightarrow {}^{4}_{2}He + {}^{4}_{2}He$$

The positron emitted are annihilated by free electrons with the production of γ-rays. The energy released in p-p chain is the same as in the C-N cycle (26.7 MeV for each Helium nucleus)

— × —

Sun: In the core,

The p-p. chain reaction occurs around 9.2×10^{37} times each second, converting about 3.7×10^{38} protons into alpha particles every second. (out of total ~ 8.9×10^{56} free protons in Sun) or about 6.2×10^{11} kg/sec.

Sun → Core (fusion), radiative zone, Tachocline, convective zone, Photosphere, Chromosphere.

The p-p chain reaction rate varies more slowly with temp, roughly as T^4 and is much more ~~paramount~~ important at lower temp. The carbon cycle rate predominates as the temp is in the vicinity of 18×10^6 K. For the case of the sun, they estimated that the rate of generation of energy in the p-p chain reaction was about temp of the sun of about 15×10^6 K.