

# **LOGARITHM AND ANTILOGARITHM CALCULATIONS**

*By: Dr. Sayanti Poddar*

Prior to the invention of calculators, logarithms were used to simplify computations in various fields of knowledge, such as navigation, surveying, astronomy, and later on, engineering.

## LOGARITHMS

- Invented by John Napier - a Scottish mathematician.
- Developed in 16<sup>th</sup> century

## LOGARITHMS

- Consider the following calculation

$$3.142 \times 12.05 \times (3.142)^2$$

This calculation is quite complex without a calculator.

One way is to do multiplication by manual way

Second way => use calculator **(NOT ALLOWED)**

Third way => use logarithmic tables

In mathematics, the **logarithm table** is used to find the value of the logarithmic function. The simplest way to find the value of the given logarithmic function is by using the **log table**. Here the definition of the logarithmic function and procedure to use the logarithm table is given in detail.

## **Logarithmic Function Definition**

The logarithmic function is defined as an inverse function to exponentiation. The logarithmic function is stated as follows

For  $x$ ,  $a > 0$ , and  $a \neq 1$ ,

$$y = \log_a x, \text{ if } x = a^y$$

Then the logarithmic function is written as:

$$f(x) = \log_a x$$

The most 2 common bases used in logarithmic functions are base  $e$  and base 10. The log function with base 10 is called the common logarithmic function and it is denoted by  $\log_{10}$  or simply  $\log$ .

$$f(x) = \log_{10}$$

The log function to the base  $e$  is called the natural logarithmic function and it is denoted by  $\log_e$ .

$$f(x) = \log_e x$$

To find the logarithm of a number, we can use the logarithm table instead of using mere calculation. Before finding the logarithm of a number, we should know about the characteristics and mantissa part of a given number

- **Characteristic Part** – The whole part of a number is called the characteristic part. The characteristic of any number greater than one is positive, and if it is one less than the number of digits to the left of the decimal point in a given number. If the number is less than one, the characteristic is negative and is one more than the number of zeros to the right of the decimal point.
- **Mantissa Part** – The decimal part of the logarithm number is said to be the mantissa part and it should always be a positive value. If the mantissa part is in a negative value, then convert into the positive value.

# LOGARITHMIC FUNCTION DEFINITION

	0	1	2	3	4	5	6	7	8	9	Mean Difference								
											1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8

## HOW TO FIND LOG OF A NUMBER?

- Consider the number 12.
- To find the logarithm of this number:
  - Step 1: Find the **characteristic**
  - Step 2: Find the **mantissa**

## FINDING CHARACTERISTIC

- Consider the number 235.7
- This can be written as  $2.357 \times 10^2$
- Therefore, the decimal point had to be shifted to the left by **2** decimal places so that the number is between 1 and 10.

2 3 5 . 7    2 . 3 5 7 x 10<sup>2</sup>

## FINDING CHARACTERISTIC

- **Step 1: Find characteristic**
- Write the number by shifting the decimal point so that the number is between 1 and 10
- Shift the decimal point and multiply by the appropriate power of 10.
- E.g. Consider the no. 12
- $12 = 12.0 = 1.2 \times 10^1$   
(shift decimal point to left by 1 place)
- The **power** of 10 is the characteristic
- Therefore, characteristic is 1

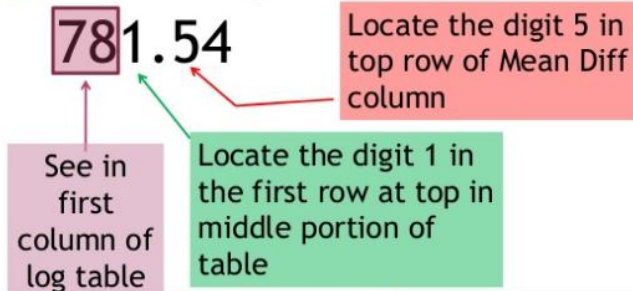
## FIND THE CHARACTERISTIC IN EACH EXAMPLE

- 78123    Characteristic ?
- 781.54    Characteristic?
- 3.142    Characteristic?
- 0.0067    Characteristic?
- 1538.2    Characteristic?

## STEP 2: FIND MANTISSA

□ Mantissa is a pure fraction, and it is found from the log tables

□ Example: Find log of 781.54



## FINDING LOG OF 781.54

□ 1: Find characteristic (Characteristic = 2)

□ 2: Find Mantissa

□ In log table, see **78** in 1<sup>st</sup> column

□ Locate the digit **1** in the first row at top

□ The number at intersection is **8927**

□ Locate digit 5 in Mean Difference column.  
The number is 3

□ **8927 + 3 = 8930**

□ **So  $\log(781.54) = 2.8930$**



# How to Use the Log Table?

The procedure is given below to find the log value of a number using the log table. First, you have to know how to use the log table. The log table is given for the reference to find the values.

- **Step 1:** Understand the concept of the logarithm. Each log table is only usable with a certain base. The most common type of logarithm table is used is log base 10.
- **Step 2:** Identify the characteristics and mantissa part of the given number. For example, if you want to find the value of  $\log_{10}(15.27)$ , first separate the characteristic part and the mantissa part.
- Characteristic Part = 15
- Mantissa part = 27
- **Step 3:** Use a common log table. Now, use row number 15 and check column number 2 and write the corresponding value. So the value obtained is 1818.
- **Step 4:** Use the logarithm table with a mean difference. Slide your finger in the mean difference column number 7 and row number 15, and write down the corresponding value as 20.

15.27

N	0	1	2	3	4	5	6	7	8	9	Mean Difference						
											1	2	3	4	5	6	7
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23
14	1431	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	7
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	9	12	6	7



- **Step 5:** Add both the values obtained in step 3 and step 4. That is  $1818+20= 1838$ . Therefore, the value 1838 is the mantissa part.

15.27

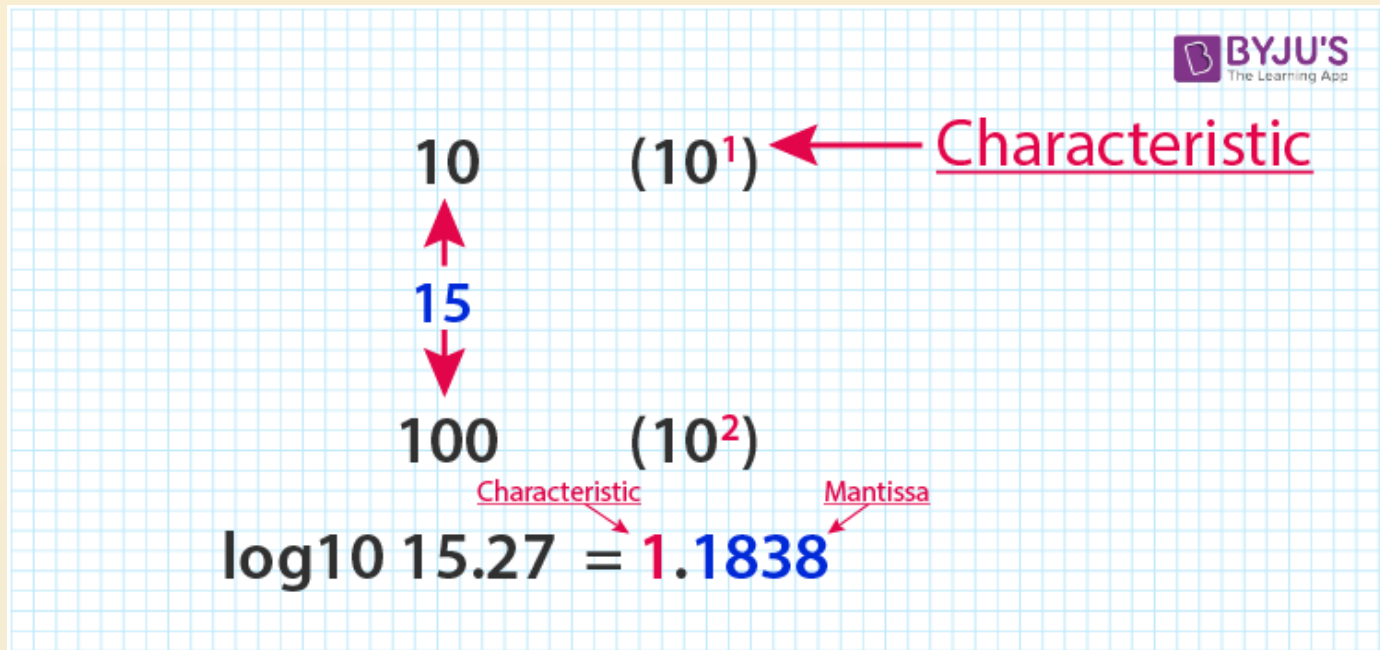
Row 15 + column 2      Row 15 + column 7

1818 + 20

= 1838

Mantissa

- **Step 6:** Find the characteristic part. Since the number lies between 10 and 100, ( $10^1$  and  $10^2$ ), the characteristic part should be 1.
- **Step 7:** Finally combine both the characteristic part and the mantissa part, it becomes 1.1838.



# Sample Example

Here the sample example to find the value of the logarithmic function using the logarithm table is given.

## Question:

Find the value of  $\log_{10} 2.872$

## Solution:

- Step 1: Characteristic Part= 2 and mantissa part= 872
- Step 2: Check the row number 28 and column number 7. So the value obtained is 4579.
- Step 3: Check the mean difference value for row number 28 and mean difference column 2. The value corresponding to the row and column is 3
- Step 4: Add the values obtained in step 2 and 3, we get 4582. This is the mantissa part.
- Step 5: Since the number of digits to the left side of the decimal part is 1, the characteristic part is less than 1. So the characteristic part is 0
- Step 6: Finally combine the characteristic part and the mantissa part. So it becomes 0.4582.

***Therefore the value of  $\log 2.872$  is 0.4582.***

## LAWS OF LOGARITHMS

$$\text{Log}(m \times n) = \text{Log } M + \text{log } N$$

$$\text{Log}\left(\frac{M}{N}\right) = \text{Log } M - \text{Log } N$$

$$\text{Log } N^M = M \text{Log } N$$

### EXAMPLE

Find  $851.9 \times 11.82$

$$\text{Log } (851.9) = 2. (9299 + 5) = 2.9304$$

$$\text{Log } (11.82) = 1. (0719 + 8) = 1.0727$$

$$+ \frac{\quad}{4.0031}$$

$$A/\text{log}(4.0031) = 1007 + 0 = 1007$$

Put decimal point after 1<sup>st</sup> digit:  $1.007 \times 10^4$

So

$$851.9 \times 11.82 = 1.007 \times 10^4$$

### EXAMPLE

Find  $1234.9 / 9.8$

$$\text{Log } (1234.9) = 3. (0899 + 17) = 3.0916$$

$$\text{Log } (9.8) = 0. (9912 + 0) = 0.9912$$

$$- \frac{\quad}{2.1004}$$

$$A/\text{log}(2.1004) = 1259 + 1 = 1260$$

Put decimal point after 1<sup>st</sup> digit:  $1.260 \times 10^2$

### EXAMPLE

Find  $1234.9 \times 9.8$

$$\text{Log } (1234.9) = 3. (0899 + 17) = 3.0916$$

$$\text{Log } (9.8) = 0. (9912 + 0) = 0.9912$$

$$+ \frac{\quad}{4.0828}$$

$$A/\text{log}(4.0828) = 1210$$

Put decimal point after 1<sup>st</sup> digit:  $1.210 \times 10^4$

So

$$1234.9 \times 9.8 = 1.210 \times 10^4$$

# Use of Antilog

## What Is an Antilog?

- An **antilog** is the result of raising the base being used to the logarithm given or calculated. Put another way, it "undoes" what calculating the logarithm of a number does and simply returns that number. In an equation of the form  $\log_b x = y$ , it is the "x" term, called the argument of the log function.
- "Antilog" can also be written  $\log_b^{-1}$  or just  $\log^{-1}$  where base 10 is implied by default.
- In summary, then:
- $\text{Antilog } x = \log_b^{-1} x = y = b^x$

A log number can then be returned to its original number. This can be done using **antilogarithm** (antilog). Thus, the antilog is the **inverse function** of log. Likewise, antilog functions to **exponentiate** a simplified log value.

To compute the antilog of a number y, you must raise the logarithm base b (usually 10, sometimes the constant e) to the power that will generate the number y.

- Here is the equation for antilog using base 10:  
 $10^x = y$   
Where x is the exponent and y is the antilog value.
- For instance, if we take this equation,  $\log(5) = x$ , its antilog will be  $10^x = 5$ .
- Log:  $\log(5) = 0.698970004336019$
- Antilog:  $10^{0.698970004336019} = 5$
- Now let's try it with a larger number.
- If we take  $\log(150,000,000,000) = x$ , its antilog will be  $10^x = 150,000,000,000$ .
- Log:  $\log_{10}(150,000,000,000) = 11.1760912590557$
- Antilog:  $10^{11.1760912590557} = 150,000,000,000$

# How to Calculate Antilog

Before finding the antilog of a number, we should know about the parts like characteristic and mantissa part.

**Characteristic Part** – The whole part is called the characteristic part. If the characteristic of any number greater than one is positive and one less than the number of digits to the left of the decimal point in a given number. If the number is less than one, its characteristic is negative and is one more than the number of zeros to the right of the decimal point.

**Mantissa Part** – The decimal part of the logarithm number for a given number is called the mantissa part and it should always be a positive value. If the mantissa part is in negative value, convert into the positive value.



# Procedure to Find the Antilog of a Number

Method 1 : Using an Antilog Table

Consider a number, 2.6452

- Step 1: Separate the characteristic part and the mantissa part. From the given example the characteristic part is 2 and the mantissa part is 6452.
- Step 2: To find a corresponding value of the mantissa part uses the antilog table. Using the antilog table, find the corresponding value. Now, find the row number that starts with .64, then the column for 5. Now, you get the corresponding value as 4416.
- Step 3: From mean difference columns find the value. Again use the same row number .64 and find the value for column 2. Now, the value corresponding to this is 2.
- Step 4: Add the values obtained in step 2 and 3, we get  $4416 + 2 = 4418$ .
- Step 5: Now insert the decimal point. The decimal point always goes the designated place. For this, you have to add 1 to the characteristic value. Now you get 3. Then add the decimal point after 3 digits, we get 441.8
- So the antilog value of 2.6452 is 441.8.

Method 2 : Antilog calculation

- Step 1 : Separate the characteristic part and the mantissa part. From the above example given, the characteristic part is 2 and the mantissa part is 6452.
- Step 2 : Know the base. For numerical computations, the base is always 10 . Therefore for computing the antilog use base 10.
- Step 3 : Calculate the  $10^x$  . x is the number which you are using. If the mantissa of the number is 0, then the computation is easy. Calculate the value  $10^{2.6452}$  . Use calculator to find the value. Finally it comes 441.7
- Both the methods produces the same result.

COMMON ANTILOGARITHM TABLE

Table with 10 columns (0-9) and 10 rows (0-9). Each cell contains a value representing the antilogarithm of the row and column indices. A 'Mean difference' column is provided at the bottom of each row.

COMMON ANTILOGARITHM TABLE

Table with 10 columns (0-9) and 10 rows (0-9). Each cell contains a value representing the antilogarithm of the row and column indices. A 'Mean difference' column is provided at the bottom of each row.

## Sample Example

Question :

Find the antilog of 3.3010

Solution:

- Given, antilog (3.3010)
- Step 1 : Characteristics part = 3 and mantissa part = 3010
- Step 2 : Use antilog table for the row .30 , then the column for 1, you get 2000.
- Step 3 : Find the value from mean difference column for the row .30 and column 0, it gives the value 0
- Step 4 : Add the values obtained in step 2 and 3 ,  $2000 + 0 = 2000$ .
- Step 5 : Now insert the decimal place. We know that the characteristic part is 3 and we have to add it with 1. Therefore we get the value 4. Insert the decimal point after 4 places, we get 2000.
- ***Therefore, the solution of the antilog 3.3010 is 2000***

- If  $\log M = x$ , then  $M$  is called the antilogarithm of  $x$  and is written as  $M = \text{antilog } x$ .
- **For example**, if  $\log 39.2 = 1.5933$ , then  $\text{antilog } 1.5933 = 39.2$ .
- If the logarithmic value of a number be given then the number can be determined from the antilog-table. Antilog-table is similar to log-table; only difference is in the extreme left-hand column which ranges from .00 to .99.

- ***Example on antilogarithm:***

1. Find antilog 2.5463.

**Solution:**

Clearly, we are to find the number whose logarithm is 2.5463. For this consider the mantissa .5463. Now find .54 in the extreme left-hand column of the antilog-table (see four-figure antilog-table) and then move horizontally to the right to the column headed by 6 of the top-most row and read the number 3516. Again we move along the same horizontal line further right to the column headed by 3 of mean difference and read the number 2 there. This 2 is now added to the previous number 3516 to give 3518. Since the characteristic is 2, there should be three digits in the integral part of the required number.

Therefore,  $\text{antilog } 2.5463 = 351.8$ .

- 2. If  $\log x = -2.0258$ , find  $x$ .

- **Solution:**

In order to find the value of  $x$  using antilog-table, the decimal part (i.e., the mantissa) must be made positive. For this we proceed as follows:

$$\begin{aligned} \log x &= -2.0258 = -3 + 3 - 2.0258 \\ &= -3 + .9742 = 3.9742 \end{aligned}$$

Therefore,  $x = \text{antilog } 3.9742$ .

Now, from antilog table we get the number corresponding to the mantissa .9742 as  $(9419 + 4) = 9423$ .

Again the characteristic in  $\log x$  is  $(-3)$ .

Hence, there should be two zeroes between the decimal point and the first significant digit in the value of  $x$ .  
Therefore,  $x = .009423$ .

## FINDING SQUARE ROOT

- Find square root of 320
- $(320)^{1/2}$
- $= \frac{1}{2} \log (320)$
- $= \frac{1}{2} \times (2.5051)$
- $= 1.2525$
- $A/\text{Log}(1.2525)$
  
- $1786 + 2 = 1788$
- $1.788 \times 10^1 = 17.88$
- **Therefore square root of 320 is 17.88**