

Function Findchain (P)  
begin

$n = \text{len}(P) - 1$

for  $i = 1$  to  $n$  do

$m[i][i] = 0$

for  $l = 2$  to  $n$  do

begin  
for  $i = 1$  to  $(n - l + 1)$  do

$j = i + l - 1$

$m[i][j] = \infty$

for  $k = i$  to  $j - 1$  do

begin  
 $q = m[i][k] + m[k+1][j] + p_i - 1, p_k, p_j$

if  $(q < m[i][j])$  then  
begin

$m[i][j] = q$

$s[i][j] = k$

end

end

end

return  $m, s$

end

Function FindOrder( $S, i, j$ )

begin

if  $(i = j)$  then

print("A",  $i$ )

else

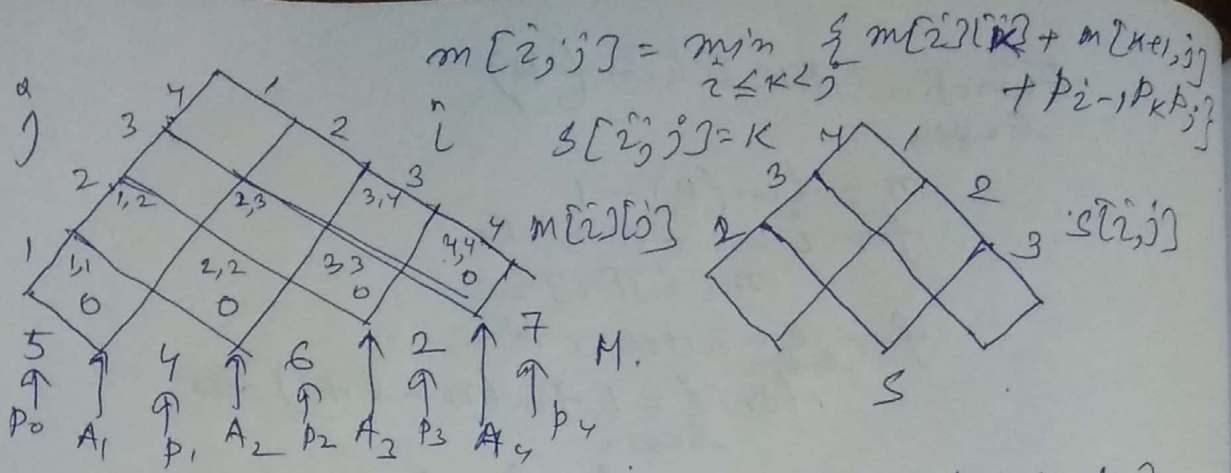
print("(", FindOrder( $S, i, s[i][j]$ ),  
FindOrder( $S, s[i][j] + 1, j$ ), ")")

end

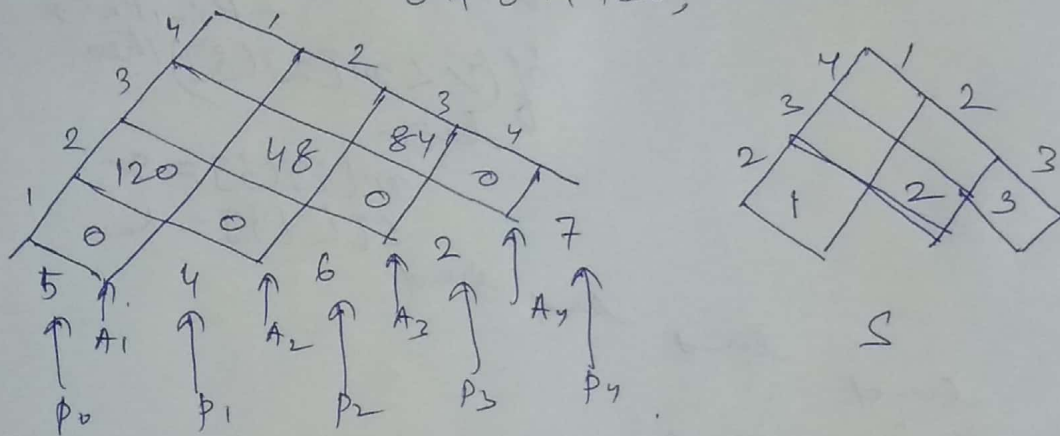
Example:  $p_0 = 5, p_1 = 4, p_2 = 6, p_3 = 2$ , and  $p_4 = 7$

Find  $m[1, 4]$  and  $s[1, 4]$ .

$(A_1 \times A_2 \times A_3 \times A_4)$



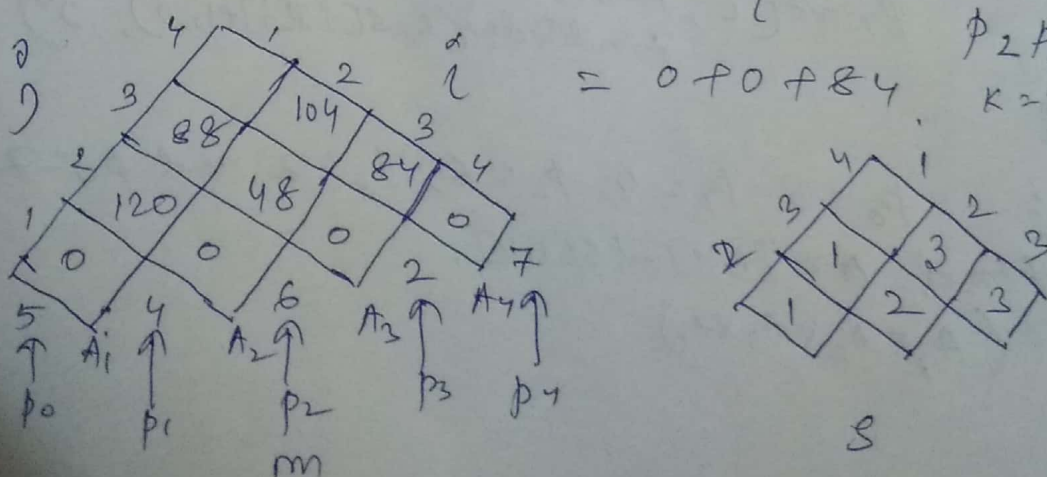
$$\begin{aligned}
 m[1][2] &= \min_{1 \leq k < 2} \{ m[1][k] + m[k+1][2] + p_{i-1} p_k p_2 \} \\
 &= \min \{ m[1][1] + m[2][2] + p_0 p_1 p_2 \} \\
 &= 0 + 0 + 120, \quad K=1
 \end{aligned}$$



~~$m[2][3]$~~

$$\begin{aligned}
 m[2][3] &= \min_{2 \leq k < 3} \{ m[2][k] + m[k+1][3] + p_{i-1} p_k p_3 \} \\
 &= \min \{ m[2][2] + m[3][3] + p_1 p_2 p_3 \} \\
 &= 0 + 0 + 48, \quad K=2
 \end{aligned}$$

$$\begin{aligned}
 m[3][4] &= \min_{3 \leq k < 4} \{ m[3][k] + m[k+1][4] + p_{i-1} p_k p_4 \} \\
 &= \min \{ m[3][3] + m[4][4] + p_2 p_3 p_4 \} \\
 &= 0 + 0 + 84, \quad K=3
 \end{aligned}$$





$$m[1][3] = \min_{1 \leq k \leq 3} \{ m[1][k] + m[k+1][3] + p_{k-1} p_k p_3 \}$$

$$= \min \begin{cases} m[1][1] + m[2][3] + p_0 p_1 p_3 \\ m[1][2] + m[3][3] + p_0 p_2 p_3 \end{cases}$$

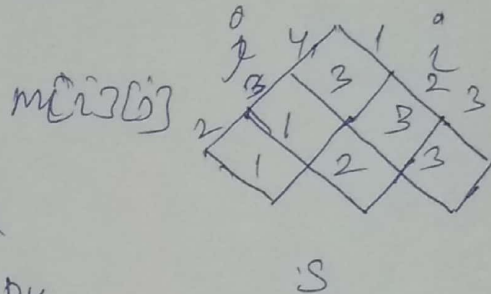
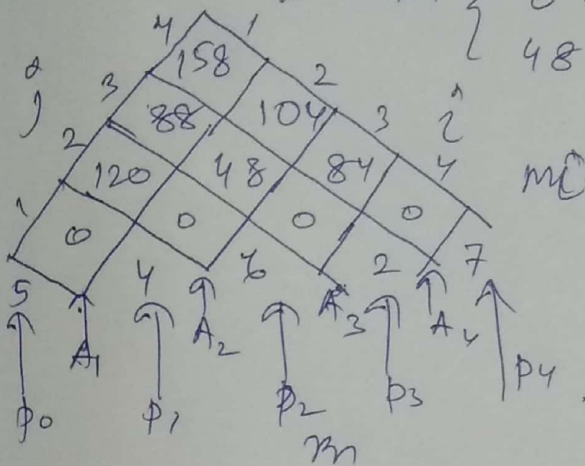
$$= \min \begin{cases} 0 + 48 + 5 \times 4 \times 2 \\ 120 + 0 + 5 \times 8 \times 2 \end{cases} = 88$$

$k=1$

$$m[2][4] = \min_{2 \leq k < 4} \{ m[2][k] + m[k+1][4] + p_{k-1} p_k p_4 \}$$

$$= \min \begin{cases} m[2][2] + m[3][4] + p_2 p_3 p_4 \\ m[2][3] + m[4][4] + p_1 p_3 p_4 \end{cases}$$

$$= \min \begin{cases} 0 + 84 + 4 \times 6 \times 7 \\ 48 + 0 + 4 \times 2 \times 7 \end{cases} = 104, k=3.$$



$$m[3][4] = \min_{1 \leq k < 4} \{ m[1][k] + m[k+1][4] + p_{k-1} p_k p_4 \}$$

$$= \min \begin{cases} m[1][1] + m[2][4] + p_0 p_1 p_4 \\ m[1][2] + m[3][4] + p_0 p_2 p_4 \\ m[1][3] + m[4][4] + p_0 p_3 p_4 \end{cases}$$

$$P(S, 1, 4)$$

$$= \min \begin{cases} 0 + 104 + 5 \times 4 \times 7 \\ 120 + 84 + 5 \times 6 \times 7 \\ 88 + 0 + 5 \times 2 \times 7 \end{cases} = 158$$

$k=3$

$$\begin{aligned} &= ("P(S, 1, 3) \cdot P(S, 4, 4)) = ("("P(S, 1, S[1][3]) P(S, S[1][3]+1, 3))" P(S, 4, 4)) \\ &= ("("P(S, 1, 1) P(S, 2, 3))" A_4) \\ &= ("("A_1 P(S, 2, 3))" A_4) = ("("A_1 ("P(S, 2, S[2][3]) P(S, S[2][3]+1, 3))" A_4) \\ &= ("("A_1 ("P(S, 2, 2) P(S, 3, 3))" A_4) A_4) \\ &= ("(A_1 (A_2 A_3)) A_4) \end{aligned}$$