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Phase transition in the Kolkata Paise Restaurant problem

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ABSTRACT

A novel phase transition behavior is observed in the Kolkata Paise Restaurant problem where a large number (N) of agents or customers collectively (and iteratively) learn to choose among the N restaurants where she would expect to be alone that evening and would get the only dish available there (or may get randomly picked up if more than one agent arrive there that evening). The players are expected to evolve their strategy such that the publicly available information about past crowds in different restaurants can be utilized and each of them is able to make the best minority choice. For equally ranked restaurants, we follow two crowd-avoiding strategies: strategy I, where each of the $n_i(t)$ number of agents arriving at the ith restaurant on the tth evening goes back to the same restaurant the next evening with probability $[n_i(t)]^{-\alpha}$, and strategy II, with probability p, when $n_i(t) > 1$. We study the steady state (t-independent) utilization fraction f : $(1 - f)$ giving the steady state (wastage) fraction of restaurants going without any customer at any particular evening. With both strategies, we find, near $\alpha_c = 0_+$ (in strategy I) or $p = 1_-$ (in strategy II), the steady state wastage fraction $(1 - f) \propto (\alpha - \alpha_c)^\beta$ or $(p_c - p)^\beta$ with $\beta \simeq$ 0.8, 0.87, 1.0, and the convergence time τ [for $f(t)$ becoming independent of t] varies as $\tau \propto (\alpha - \alpha_c)^{-\gamma}$ or $(p_c - p)^{-\gamma}$, with $\gamma \simeq$ 1.18, 1.11, 1.05 in infinite-dimensions (rest of the $N - 1$ neighboring restaurants), three dimensions (six neighbors), and two dimensions (four neighbors), respectively.

Social games where the players or agents try to choose the less crowded or minority solutions (to avail the scarce resources) are very common. In such games, a macroscopically large number of agents make decision parallelly and iteratively (in the absence of any dictator), based on publicly available information (regarding past mistakes and successes), to choose where she can be alone (and avail the resource). Eventually, such collective learning makes it socially efficient. One such social minority game is the Kolkata Paise Restaurant (KPR) problem. We study the steady state statistics and the phase transition behavior of the KPR problem.[1,2] KPR is a many-agent and many-choice repeated game, where the agents collectively learn from past mistakes and how to share best the limited resources. In this kind of game, each agent tries to anticipate and choose her own strategy every time (learning from the publicly available past information) in parallel mode (unguided; in the absence of any instruction or non-playing agent/dictator). The restaurants are assumed to prepare fixed meal plates every evening, which are equally priced (hence no budget constraint for customers). Only the crowd avoidance abilities determine the individual's success in securing a meal on any or successive evenings. We show that in KPR, two stochastic strategies can eventually lead to the most efficient social solution at some limiting control parameter values, corresponding to a phase transition point, thereby implying a very long convergence time (critical slowing down).

I. INTRODUCTION

We consider here the case of N restaurants and N agents or customers or players who decide every evening (on the basis of information about the past evenings, available to everyone) which