

2020

STATISTICS — HONOURS — PRACTICAL

Paper : CC-11P

Full Marks : 30

*The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.*

1. Let X and Y be a random sample from the Poisson distribution with unknown mean θ . In order to test $H_0 : \theta = 1$ against $H_1 : \theta > 1$, the following procedure is suggested :

If $X \leq 1$, then do not reject H_0 and if $X \geq 4$, then reject H_0 .

If X equals 2 or 3, then reject H_0 if $X + Y \geq 4$; otherwise, do not reject H_0 .

- (a) Find the size of the above test. Suppose it is δ .
- (b) Find the power of the above test at $\theta = 2$.
- (c) Find the power (at $\theta = 2$) of the uniformly most powerful test (based on X and Y) with size δ for testing $H_0 : \theta = 1$ against $H_1 : \theta > 1$ and comment.
- (d) Suggest another test of size δ and find the power of your test at $\theta = 2$. 4+3+4+3
2. The probability for any student to pass a Statistics paper in the k -th attempt is $P(X = k) = \theta(1 - \theta)^{k-1}$, where $k = 1, 2, 3, \dots$, and θ ($0 < \theta < 1$) is an unknown parameter. For a random sample of 42 students, the arithmetic mean of the number of attempts to pass the above paper is observed to be 4. Find the maximum likelihood estimate (MLE) of θ . Find the maximum value of the likelihood function. Is MLE unique here? Justify your answer. Also find (with justification) the MLE of $P(X < 3)$. 3+2+1+2
3. The probability density function f of a random variable X is given by

$$f(x) = \frac{2(\theta - x)}{\theta^2} \text{ if } 0 \leq x \leq \theta,$$

and $f(x) = 0$, otherwise. Here θ is positive and unknown.

A single observation, say X , is drawn at random from this distribution.

Let $[aX, bX]$ be a confidence interval for θ , where $a < b$. What values of b , if any, will ensure a confidence coefficient 0.95 for the above confidence interval when (i) $a = 5$, (ii) $a = 0.5$ and (iii) $a = 1$.

Find the expected lengths of the confidence intervals you have obtained and comment. 2+2+1+(2+1)